# Cities’ Demand-Driven Industrial Compositions and Refined Market-Size Effects*

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**Abstract**

Large cities specialize in income-elastic sectors. I develop a model of cities with sector-specific trade costs and income elasticities of demand to explain this finding and derive potential implications. Higher productivity and better amenities make cities larger, command agglomeration economy and higher income, and, consequently, spend more on income-elastic sectors. This demand pattern translates into a more pronounced production pattern, explaining the observed employment distribution. The model refines the market-size effect on factor prices, showing that wages rise with city (or country) size and expenditure shares in high-tradability sectors. When sectors are gross complements or high-tradability sectors are income-elastic, it suggests that large cities spend more on high-tradability sectors, raising the city-size wage premium.

**Keywords:** Home-market effect, Market size, Trade cost, Non-homothetic preference, Cities

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# 1 Introduction

Industrial composition varies significantly across cities. For example, Detroit and Silicon Valley are synonymous with cars and computers, respectively. Figure 1 shows that much of the variation in employment composition across the U.S. Metropolitan Statistical Areas (MSAs) is linked to the income elasticity of demand for that industry's output. Industries that employ relatively more workers in large MSAs have high-income elasticities of demand, such as air transport services and recreational and other services. Conversely, industries that employ relatively more workers in small MSAs, such as beverage and tobacco products, have lower income elasticities.

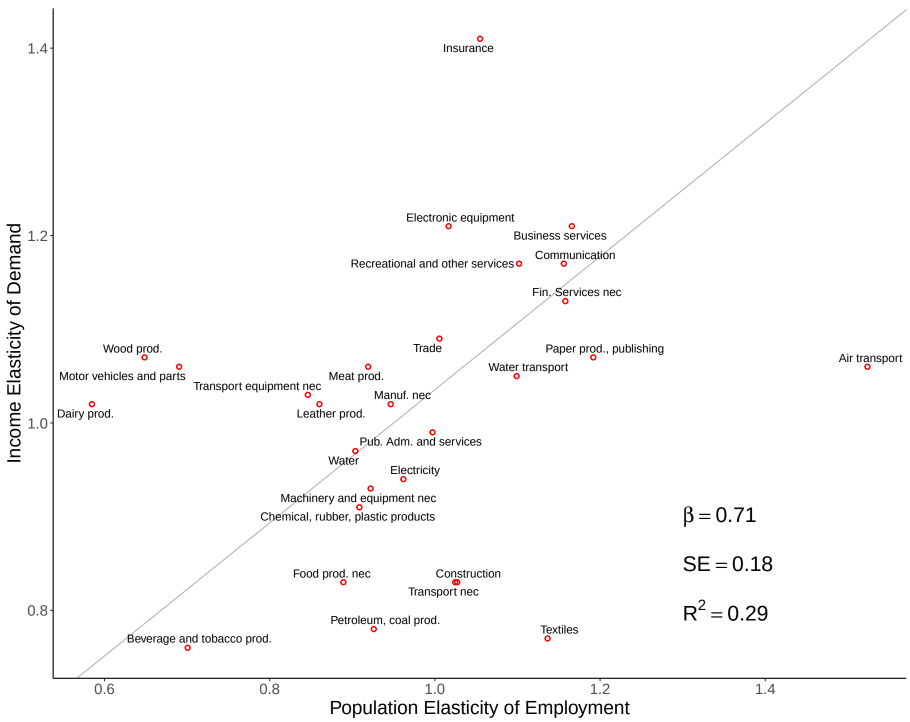


Figure 1: Elasticity of Employment with respect to MSA's Population and Elasticity of Demand with respect to Income

*Notes:* The upward-sloping line represents the weighted regression line, with total sectoral employment in the sample used as weights. This graph omits the "Forestry" sector, whose vertical and horizontal axis values are 0.36 and 0.19, respectively, with a weight of less than 0.1 percent. Standard errors are calculated using the bootstrap method. MSA population data are obtained from the 2016 Bureau of Economic Analysis. Sectoral employment is calculated by mapping the 2016 County Business Patterns data, based on NAICS, to GTAP sectors. The population elasticity of employment is obtained while controlling for regions $\in$ {Northeast, Midwest, South, West}. Income elasticity estimates are from Caron et al. (2020). For data construction details, see Appendix A.

Motivated by this new finding, this study explores the implications of cross-location demand patterns by developing a model of many equidistant cities with many sectors that feature sector-specific trade costs and income elasticities of demand. My model builds on Matsuyama (2019), who theoretically studies international trade patterns with a continuum of differentiated-goods sectors under monopolistic competition and heterogeneous sectoral income elasticities. His model endogenously generates a demand pattern that a large or productive country spends more on income-elastic sectors, which translates into employment and trade patterns. I extend Matsuyama (2019)'s model by introducing worker mobility, location-specific amenity levels, sector-specific trade costs, and more than two locations.

In my model, the fundamentally different productivity levels and amenities of cities generate a pattern of city size, eventually leading to the specialization pattern consistent with Figure 1. Cities with better fundamentals attract workers, generating an agglomeration economy. Owing to the fundamentals and agglomeration economy, residents in large cities have higher real income, spending relatively more on income-elastic industries. This expenditure pattern yields an employment pattern, driven by the home-market effect, initially hypothesized by Linder (1961) and formally theorized by Krugman (1980). When the relative market size of sectors varies across regions, regions export goods for which they have relatively large domestic markets. In the presence of trade costs and increasing returns, local firms are incentivized to operate in a sector with a relatively larger home market, and this incentive is strong enough to amplify the demand pattern into the production pattern. Hence, in equilibrium, regions with better fundamentals become larger and specialize in income-elastic sectors.

As a new theoretical result, I show that sectoral expenditure composition influences wages in cities and countries, refining the market-size effect on factor prices. Krugman (1980, 1991) demonstrates that the larger country commands the higher wage because an advantage in market size—savings of trade costs—must be offset by a disadvantage in production cost—a higher wage—due to firms' free entry.[1] Conversely, given a wage level, a larger market generates stronger labor demand by attracting firms. In my model, this effect strengthens in sectors with lower trade costs (higher tradability), reflecting their intensive inter-city

[1] The definitions and terminology of the market-size effect and the home-market effect have some variations in the literature. This paper uses the "market-size effect on factor prices" and the "home-market effect on trade patterns" or simply the "home-market effect." It also calls them the "market-size effects."

competition. Consequently, shifting expenditures toward higher-tradability sectors produces greater city-level aggregate labor demand, raising the wage. A city-size wage premium potentially reflects two effects through this newfound channel. First, when high-tradability sectors are income-elastic, large cities spend more on them, raising the premium. Second, inter-sectoral substitution strengthens and weakens the market-size effect on factor prices when sectors are gross complements and substitutes, respectively, affecting the premium correspondingly. I discuss these effects on an economy consisting of manufacturing and services, which have substantially different trade costs and income elasticities.

Understanding cities' industrial composition is important in its own right from several perspectives. First, it is a critical factor for local economies because local economic performance is significantly affected by the industries located in a city (e.g., Autor et al. 2013; Mian and Sufi 2014). Second, industrial composition is related to heterogeneous returns to experience across cities. Eckert et al. (2022) study a natural experiment of refugees arriving in Denmark and document 35% faster wage growth with each additional year of experience for refugees. They find that a substantial part of this growth difference is attributable to cities' industrial composition, suggesting that understanding the determinants of industrial composition is essential in studying productivity growth in cities. Third, the mechanism that drives industrial composition is also crucial for researchers who want to exploit regional variation in the size of industries. In this study, I show that industrial composition is related to city size. Given this relationship, regressing dependent variables on one or two independent variables (e.g., industrial size, city size, or wage levels) while not controlling for the rest of the examined locations might lead to an omitted variable bias problem. Understanding the mechanism can help researchers avoid this endogeneity issue.

To the best of my knowledge, this study is the first on cross-city inter-sectoral specialization patterns within differentiated-goods sectors from a demand-side perspective. On intra-city issues, Hoelzlein (2023) studies firm and labor sorting in his quantitative model with heterogenous income elasticities and skill intensities. Most of the existing works on cross-city specialization patterns focus on non-demand side factors (Behrens and Robert-Nicoud 2015; Davis and Dingel 2019; Duranton and Puga 2005; Gaubert 2018; Henderson and Ono 2008). A few studies focus on the demand side's effect on cross-city differences (e.g., Handbury 2019). Among these studies, the one most closely resembling mine is by Dingel (2017). However, there are two significant differences between Dingel (2017) and this study. First, we focus on different types of specialization and trade patterns. Dingel (2017) examines intra-sectoral trade, where different quality goods are gross substitutes. In contrast, this study examines inter-sectoral specialization, where goods in different sectors can be gross complements or substitutes. Second, my model has mobile agents, unlike Dingel (2017)'s model. These assumptions fit the urban economy environment, enabling the analysis of the relationship between the size of a city and its industrial composition.

The new result on factor prices contributes to studies of income inequality across regions and countries. Empirical works that analyze the market-size effect consider the proximity to and the size of other markets as well as home-market size. The aggregate measure is known as market potential (Harris (1954); Hanson (2005)) or market access (Redding and Venables (2004)), and many have estimated the effect on wages or economic development (Hanson (1997); Redding and Venables (2004); Hanson (2005); Head and Mayer (2006); Redding and Sturm (2008); Head and Mayer (2011); Brülhart et al. (2012); Jacks and Novy (2018); Jaworski and Kitchens (2019)); however, they assume a single goods-producing sector in constructing the market potential. Although it is practically reasonable, this study suggests that models with multiple goods-producing sectors and location-specific expenditure composition (this variation is significant, especially across countries) can improve quantitative analysis in future research.

Furthermore, this work contributes to two strands of the international trade literature. One focuses on demand variation as a driver of trade or industrial concentration (e.g., Flam and Helpman 1987; Stokey 1991; Fajgelbaum et al. 2011; Caron et al. 2014; Matsuyama 2019; Costinot et al. 2019); the other examines that of variation in tradability (Amiti (1998); Davis (1998); Hanson and Xiang (2004); Behrens (2005); Laussel and Paul (2007); Erhardt (2017)). My model produces a condition for a location to become a net exporter in a given sector; it simultaneously reflects the two factors, refining our understanding of the home-market effect.

This paper's technical contribution is the simultaneously obtained analytical results on city size, wages, and trade patterns in the presence of increasing returns, trade costs, worker mobility, and rich sectoral characteristics in many equidistant cities. I extend the technique of Zeng and Uchikawa (2014), who examine a model of many equidistant countries, to prove the existence of an equilibrium.[2] Subsequently, I obtain

[2] Redding (2016) provides a condition for a spatial equilibrium to exist and be unique, as well as comparative statics in a model with a rich geography and increasing returns, following Allen and Arkolakis (2014). However, multiple

sufficient conditions on cities' productivity—jointly with amenities in the case of a common trade cost—for a city to become larger and, separately, to command a higher wage than others. Finally, I analyze inter-city trade patterns.

The rest of this paper is organized as follows. Section 2 introduces the model. Section 3 characterizes the equilibrium conditions by obtaining city-level labor demand and labor supply as functions of the city's factor price. The labor demand takes expenditure composition and population allocation as given and exhibits the market-size effect on factor prices; thus, this part independently applies to a broad range of models. Section 4 obtains intuitions for city-size and factor-price patterns using these equilibrium conditions for one city. Section 5 provides cross-city analysis in equilibrium with common tradability and replicates the employment pattern of Figure 1. Subsequently, Section 6 completes the cross-city analysis with sector-specific tradability and discusses potential implications for an economy consisting of manufacturing and services. Subsequently, Section 7 presents two robustness checks of the employment pattern of Figure 1, one of which a model prediction guides. Finally, Section 8 concludes the paper.

# 2 The Model

In this section, I introduce a model of $N$ cities, whose set is $\mathcal{N}$, and $K$ sectors, whose set is $\mathcal{K}$. Sectors differ in relative real income elasticities $\epsilon(k)$ and iceberg trade costs $\tau_k$, as well as preference shifters $\beta_k$ and parameters for variable and fixed costs, $v_k$ and $f_k$. Subscripts $n$, $i$, and $j$ denote cities, and $k$ and $\ell$ denote sectors. Cities fundamentally differ in productivity $\lambda_n$ and amenity level $a_n$. A mass of $L$ workers are freely mobile and homogeneous except for an inherent preference for cities. Conditional on location, individual labor supply is inelastic. I start by explaining the workers' problem. Appendix B provides detailed derivations and proofs.

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sectors with a non-Cobb-Douglas upper-tier preference prohibit us from applying his technique to my model.

### Workers' Problem

The problem for a worker $\zeta$ is given by

$$\max_{n\in\mathcal{N},C_n,\{Q_{nk}\}_{k\in\mathcal{K}},\{q_{nk}(\nu)\}_{\nu\in\Omega_{nk},k\in\mathcal{K}}} C_n \cdot a_n \cdot \delta(\zeta, n), \tag{1}$$

$$s.t.\ C_n = \left[\sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}},$$

$$\forall k\ Q_{nk} = \left[\int_{\Omega_{nk}} q_{nk}(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}},$$

$$E_n = \sum_{k\in\mathcal{K}} \int_{\Omega_{nk}} p_{nk}(\nu) q_{nk}(\nu) d\nu,$$

where $n$ is the city chosen to reside in, $p_{nk}(\nu)$ and $q_{nk}(\nu)$ are the price and consumption, respectively, of variety $\nu$ in sector $k$ in city $n$, $\Omega_{nk}$ and $Q_{nk}$ are the set of available varieties and the composite consumption, respectively, of sector $k$ in city $n$, $E_n$ is the nominal income in city $n$, $a_n > 0$, $0 < \eta < \sigma$, $\eta \neq 1$, $\min_{k\in\mathcal{K}}\{\epsilon(k)/(1-\eta)\} > -1$, and $\sigma > 1$. The utility comprises three factors: $C_n$, $a_n$, and $\delta(\zeta, n)$.

The functional form of real income $C_n$ captures the consumer's non-homothetic preference, following Comin et al. (2021), Hoelzlein (2023), and Matsuyama (2019). When $\epsilon(k) = 0$ for all $k \in \mathcal{K}$, this $C_n$ becomes a standard homothetic constant elasticity of substitution function. When $\epsilon(k)$ varies across sectors, a higher $\epsilon(k)$ corresponds to a more income-elastic sector; the weight $\beta_k^{1/\eta} C_n^{\epsilon(k)/\eta}$ becomes relatively more influential as real income grows. When solving the consumption optimization problem, a convenient property of the functional form of $C_n$ becomes clearer. The demand function derived from this preference becomes

$$Q_{nk} = \beta_k C_n^{\epsilon(k)} P_{nk}^{-\eta} C_n^{1-\eta} E_n^{\eta}, \tag{2}$$

where $P_{nk} = \left[\int_{\nu\in\Omega_{nk}} p_{nk}(\nu)^{1-\sigma} d\nu\right]^{1/(1-\sigma)}$ is the price index for sector $k$ in city $n$. This demand function shows that the relative real income elasticity of demand $\epsilon(k)$ and price elasticity $\eta$ at the sector level are separated. I refer to $\epsilon(k)$ simply as the income elasticity throughout the paper. Price elasticities are common among sectors, and price elasticity at the sector level is lower than that at the variety level ($\eta < \sigma$). Eq. (2) implies that sectors are gross complements when $\eta < 1$ and gross substitutes when $\eta > 1$. Additionally, the expenditure share of sector $k$ is obtained as follows:

$$s_{nk} \equiv \frac{P_{nk}Q_{nk}}{\sum_{\ell \in \mathcal{K}} P_{n\ell}Q_{i\ell}} = \beta_k C_n^{\epsilon(k)} \left(\frac{P_{nk}}{P_n}\right)^{1-\eta}, \tag{3}$$

where $P_n = \left(\sum_{k\in\mathcal{K}} \beta_k C_n^{\epsilon(k)} P_{nk}^{1-\eta}\right)^{\frac{1}{1-\eta}}$, the price index of city $n$, is defined by $P_n C_n = E_n$. This expenditure share is log-supermodular in $C_n$ and $\epsilon(k)$ ($\partial^2 \log s_{nk}/\partial C_n \partial \epsilon(k) > 0$).[3] It shows that, holding the price indices $\{P_k\}_{k\in\mathcal{K}}$ constant, agents with higher real income $C_n$ spend relatively more on sectors with high $\epsilon(k)$. The expense of this convenient property is endogenously varying returns to scale. The elasticity of $C_n$ with respect to composite consumption $Q_{nk}$ is $s_{nk}[1 + \sum_{k\in\mathcal{K}} s_{nk}\epsilon(k)/(1-\eta)]^{-1}$. Thus, a uniform proportional change of $\{Q_{nk}\}_{k\in\mathcal{K}}$ does not generate an equal proportional change of $C_n$ unless $\sum_{k\in\mathcal{K}} s_{nk}\epsilon(k)/(1-\eta) = 0$. Furthermore, it requires $\min_{k\in\mathcal{K}}\{\epsilon(k)/(1-\eta)\} > -1$ to ensure the global monotonicity of $C_n$.

Workers homogeneously appreciate amenities offered by city $n$, such as weather, landscape, and historic heritage, the overall level of which $a_i$ measures. While "amenity" generally refers to access to local services and consumer goods (e.g., restaurants)—referred to as consumption amenities—in this model, those local services and goods contribute to $C_n$ when consumed.[4]

Additionally, workers have heterogeneous preferences for cities (following Tabuchi and Thisse (2002), Redding (2016), and others); the worker $\zeta$ and city $n$ pair receives the idiosyncratic utility shock $\delta(\zeta, n)$. I assume $\delta(\zeta, n)$ is independent and identically distributed (i.i.d.) across workers and cities according to the Fréchet distribution with shape parameter $1/\gamma$ ($Pr[\delta < x] = e^{-x^{-1/\gamma}}$). Workers choose the city that offers the highest utility, considering their consumption optimization. Thus, given the products of real income and the utility from amenities in cities, $\{C_n a_n\}_{n\in\mathcal{N}}$, the probability of choosing city $n$ for a given agent $\zeta$ is derived as $Pr[a_n C_n \delta(\zeta, n) = \max_i a_i C_i \delta(\zeta, i)] = (a_n C_n)^{1/\gamma} / \sum_{i\in\mathcal{N}} (a_i C_i)^{1/\gamma}$. As the shock is i.i.d., city $n$'s population is as follows:

$$L_n = \frac{(C_n a_n)^{1/\gamma}}{\sum_{i\in\mathcal{N}} (C_i a_i)^{1/\gamma}} L. \tag{4}$$

[3] A differentiable function $f(x, y)$ is log-supermodular in $x$ and $y$ if and only if $\partial^2 \log f(x, y)/\partial x \partial y > 0$.

[4] My model omits non-tradable sectors and classifies local services and goods as sectors with extremely high but not unbounded trade costs. They are arguably not completely non-tradable as travelers can visit restaurants, for example.

Given real income and the utility from amenities in cities, the lower $\gamma$ is, the greater the population dispersion; therefore, the parameter $\gamma$ measures the dispersion force from heterogeneous preferences. This dispersion force is crucial in generating a cross-city demand pattern by allowing real income to vary across cities in equilibrium.[5]

### Firms' Problem

Production in my model is based on Krugman (1980). For all sectors, there are endogenous sets of varieties, homogeneous firms, and monopolistic competition. I let $w_n$ denote the wage in city $n$. Cities differ in productivity $\lambda_n$; each firm in sector $k$ in city $n$ must employ $f_k/\lambda_n$ units of labor as a fixed cost and $v_k/\lambda_n$ as a variable cost to produce a variety. A shipment of a variety from city $n$ to city $i$ requires an iceberg trade cost $\tau_{nik} < \infty$. To keep the model tractable for analytical results, I simplify geography. Cities' locations are symmetric, and the iceberg trade cost is constant and greater than one between different cities, conditional on a sector; that is, $\tau_{nik} = \tau_k > 1$ when $n \neq i$, and $\tau_{nik} = 1$ when $n = i$. The problem for a firm that produces variety $\nu$ in sector $k$ in city $n$ is

$$\pi_{nk}(\nu) = \max_{\{p_{nik}(\nu), q_{nik}(\nu)\}_{i\in\mathcal{N}}} \sum_{i\in\mathcal{N}} \left[ p_{nik}(\nu) q_{nik}(\nu) - \{\tau_k + \mathbb{1}\{i = n\}(1-\tau_k)\} \frac{v_k q_{nik}(\nu)}{\lambda_n} w_n \right] - \frac{f_k}{\lambda_n} w_n, \tag{5}$$

$$s.t. \forall i,\ q_{nik}(\nu) = p_{nik}(\nu)^{-\sigma} P_{ik}^{\sigma} Q_{ik},$$

where $\pi_{nk}(\nu)$ is the profit from optimized production, $p_{nik}(\nu)$ is the market price for city $i$, $q_{nik}(\nu)$ is the market quantity for city $i$, and $\mathbb{1}\{i = n\}$ is an indicator function taking the value of one when $i = n$. In the following part, I omit $\nu$ unless necessary. If sector $k$ in city $n$ has non-zero production in equilibrium, $\pi_{nk}$ must be zero such that there are no new entrants, which is the zero-profit condition. I focus on equilibria in which all cities have non-zero production in all sectors, following Matsuyama (2019) and Hanson and Xiang

[5] The model can incorporate workers' land consumption paired with inelastic land supply as an additional dispersion force using the same functional form as Hoelzlein (2023). In such a case, real income consists of goods and land consumption. However, without heterogeneous preferences, real income equalizes across cities, eliminating demand patterns. Thus, heterogeneous preferences are necessary even with the dispersion force from inelastic land supply.

(2004). The definition of such an equilibrium is as follows.

**Definition of Equilibrium**

An equilibrium is $\{L_n, C_n, w_n, E_n, P_n\}_{n\in\mathcal{N}}$, $\{p_{nik}, q_{nik}\}_{(n,i,k)\in\mathcal{N}^2\times\mathcal{K}}$, and $\{\Omega_{nk}\}_{(n,k)\in\mathcal{N}\times\mathcal{K}}$ such that

1. workers optimize consumption and locational choice as eq. (1) for $n \in \mathcal{N}$,

2. workers' income is given by $E_n = w_n$ for $n \in \mathcal{N}$,

3. producers optimize production as eq. (5) for all $k \in \mathcal{K}$ and $n \in \mathcal{N}$,

4. the zero-profit condition holds such that $\pi_{nk} = 0$ for all $k \in \mathcal{K}$ and $n \in \mathcal{N}$,

5. the national labor market clearing condition $\sum_{n\in\mathcal{N}} L_n = L$ holds, and

6. the local labor market clearing condition $\sum_{k\in\mathcal{K}} x_{nk} = 1$,
   where $x_{nk} \equiv \int_{\Omega_{nk}} \left(\sum_{i\in\mathcal{N}} \{\tau_k - \mathbb{1}\{i = n\}(\tau_k - 1)\}\, v_k q_{nik}/\lambda_n + f_k/\lambda_n\right) d\nu/L_n$ is the employment share of sector $k$ in city $n$, holds for all $n \in \mathcal{N}$.

# 3 Equilibrium Conditions

This section characterizes two equilibrium conditions: city-level labor demand and city-level labor supply. The city-level labor demand shows the effect of market size on factor prices and that of expenditure composition. The analysis of these effects is applicable to models with immobile workers (i.e., international trade models) and models with homothetic preference because it takes population allocation and expenditure composition as given.

**Sectoral Labor Demand**

I derive sectoral labor demand from the zero-profit conditions. Given the well-known optimized prices $p_{nnk} = \sigma v_k w_n/(\sigma - 1)\lambda_n$ and $p_{nik} = \tau_k v_k w_n/(\sigma - 1)\lambda_n$ for $i \neq n$, the profit for a firm in sector $k$ in city

$n$ becomes

$$\pi_{nk} = \left( \frac{1}{\sigma - 1} \frac{v_k}{\lambda_n} q_{nnk} + \tau_k \sum_{i \neq n} \frac{1}{\sigma - 1} \frac{v_k}{\lambda_n} q_{nik} - \frac{f_k}{\lambda_n} \right) w_n. \tag{6}$$

I set $v_k = (\sigma - 1)/\sigma$ and $f_k = 1/\sigma$ by replacing $\beta_k$ with $\tilde{\beta}_k = \beta_k \left( f_k^{\frac{1}{\sigma-1}} v_k \sigma^{\sigma/(\sigma-1)} (\sigma - 1)^{-1} \right)^{1-\eta}$ without loss of generality, exploiting the isomorphism.[6] Then, the zero-profit condition ($\pi_{nk} = 0$) becomes

$$1 = L_n \left( \frac{w_n}{\lambda_n} \right)^{-\sigma} P_{nk}^{\sigma-1} E_n s_{nk} + \tau_k \sum_{i \neq n} L_i \left( \tau_k \frac{w_n}{\lambda_n} \right)^{-\sigma} P_{ik}^{\sigma-1} E_i s_{ik}.$$

This zero-profit condition applies to $N$ cities. Matrix operations yield

$$1 = (1 - \phi_k) L_n E_n s_{nk} P_{nk}^{\sigma-1} \left( \frac{w_n}{\lambda_n} \right)^{-\sigma} + \frac{\phi_k N}{1 + \phi_k (N - 1)} \left( \frac{\sum_{i \in \mathcal{N}} \left( \frac{w_i}{\lambda_i} \right)^{\sigma}}{N} \right) \left( \frac{w_n}{\lambda_n} \right)^{-\sigma}, \tag{7}$$

where $\phi_k (\equiv \tau_k^{1-\sigma} < 1)$ measures the tradability of sector $k$ and is often called "freeness of trade" in the "new trade theory" literature (Baldwin et al. 2003). Eq. (7) equates fixed cost, normalized to one, on the left-hand side to gross profit, defined as profit before deducting fixed cost, on the right-hand side.

Eq. (7) decomposes a firm's gross profit into two sources. The first source is the market access exclusive to local firms, reflected in the first term on the right-hand side of eq. (7).[7] The size of this exclusive market access increases with the sectoral market size $L_n E_n s_{nk}$; thus, a larger sectoral market, ceteris paribus, raises profit for local firms. The factor $P_{nk}^{\sigma-1}$ is an inverse measure of competition intensity; more competitors and lower prices of other firms' varieties reduce profit. The second term summarizes the gross profit from the "nationwide market"—the summation of a fraction $\phi_k$ of every market, which firms can "access" from

[6] This normalization is common in the "new trade theory" and "new economic geography." See box 2.2. of Baldwin et al. (2003) for an additional explanation of why it is without loss of generality.

[7] $(1 - \phi_k) L_n E_n s_{nk} P_{nk}^{\sigma-1}$ corresponds to the market access (Redding and Venables (2004)) that only local firms have and that of the real market potential (Head and Mayer (2004). Firms outside city $n$ need to pay trade costs to sell their varieties in the city $n$ market; they raise the price by a factor of $\tau_k$ and receive profit that decreases by a factor of $1 - \phi_k$. One can interpret this as firms selling varieties at factory gate price $(\sigma v_i w_i / (\sigma - 1) \lambda_i)$ have access to the fraction $\phi_k$ of the city $n$ market. Conversely, local firms have the exclusive market access $(1 - \phi_k) L_n E_n s_{nk}$.

any city. A higher value of $w_n/\lambda_n$ reduces profit from both sources, as it translates into a higher price. Throughout the paper, I refer to this variable as the factor price, interpreting effective labor ($\lambda_n L_n$) as the only factor in a city. This metric captures the productivity-adjusted compensation that firms face across different locations. In the nationwide market, a factor price is evaluated against the nationwide average—the power mean $\left[\sum_{i\in\mathcal{N}}(w_i/\lambda_i)^\sigma/N\right]^{1/\sigma}$. Thus, eq. (7) implies advantages and disadvantages in these three dimensions—market size, competition intensity, and factor prices—must offset each other in equilibrium.

Given a city's factor price, a larger sectoral market attracts firms, intensifying competition. I translate this intensive competition into labor demand. I rewrite eq. (7) as

$$(1-\phi_k)L_n E_n s_{nk} P_{nk}^{\sigma-1}\left(\frac{w_n}{\lambda_n}\right)^{-\sigma} = \frac{1}{1+\mu_{nk}}, \tag{8}$$

where

$$\mu_{nk} = \left[\frac{(\phi_k^{-1}+N-1)(w_n/\lambda_n)^\sigma}{\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma} - 1\right]^{-1} > 0. \tag{9}$$

The right-hand side $(1+\mu_{nk})^{-1}$ is the share of the exclusive market assess in gross profit. The normalization of $f_k = 1/\sigma$ makes labor demand by a firm equal $1/\lambda_n$. Subsequently, $P_{nk}^{1-\sigma} = (\lambda_{nk}x_{nk}L_n)\cdot(w_n/\lambda_n)^{1-\sigma} + \sum_{n\neq i}(\lambda_{ik}x_{ik}L_i)\cdot(\tau_k w_i/\lambda_{ik})^{1-\sigma}$. After substituting this expression into eq. (8), matrix operations yield

$$x_{nk}L_n w_n = (1+\mu_{nk})s_{nk}L_n w_n - \frac{(w_n/\lambda_n)^\sigma}{\phi_k^{-1}+N-1}\sum_{i\in\mathcal{N}}\frac{L_i w_i}{(w_i/\lambda_i)^\sigma}(1+\mu_{ik})s_{ik}. \tag{10}$$

Eq. (10) shows two forces on the relationship between sectoral market size $s_{nk}L_n w_n$ and sectoral labor demand in value $x_{nk}L_n w_n$.

First, sectoral labor demand increases with sectoral market size more than one-for-one. The difference in expenditures, ceteris paribus, is amplified to that in labor demand because $\mu_{nk} > 0$. Thus, the endogenous variable $\mu_{nk}$ measures the amplification strength, and I call it the *"home-market multiplier"* hereafter. This amplification occurs because of inter-city competition. Eq. (8) implies that $s_{nk}P_{nk}^{\sigma-1}$ is constant, given a city and tradability; thus, a higher expenditure share must intensify competition proportionally. However, a proportional increase of local firms does not suffice because the market has competitors from other cities, diluting the impact of local entrants on the price index. Consequently, a disproportionate amount of firms enter the market until the profit becomes zero. Furthermore, this excessive amount of firms increases net exports, which is $(x_{nk} - s_{nk})L_n w_n$ in value. Thus, $\mu_{nk}$ also measures the strength of the sectoral home market as a net-export driver.

Second, higher tradability strengthens this amplification because high $\phi_k$ raises the home-market multiplier. This might sound counterintuitive because low trade costs weaken the incentive for firms to locate in larger markets to save on trade costs—the incentive that is central to the new trade theory and new economic geography. Indeed, a large $\phi_k$ shrinks the exclusive market access in eq. (8), reflecting this effect. However, there are two more effects. First, recall $(1+\mu_{nk})^{-1}$ is the share of the exclusive home-market access in gross profit, which decreases in $\mu_{nk}$. Higher tradability improves firms' access to other markets. Consequently, firms make greater gross profit in the nationwide market, recovering a significant portion of fixed costs there. Conversely, high tradability escalates competition in the home market, ensuring the aggregate gross profit does not exceed the fixed cost. Consequently, intensifying the competition further—raising $P_{nk}^{1-\sigma}$—requires more entrants. Second, higher tradability transforms local entrants into more significant competitors for firms in other cities. Thus, changing the relative intensity of competition, measured by $P_{nk}^{1-\sigma}/P_{ik}^{1-\sigma}$, requires more entrants. These two effects dominate the diminishing incentive to save trade costs, generating stronger labor demand.[8] This sector-specific amplification strength makes expenditure composition affect cities' factor prices, which I analyze in the next subsection.

### City-Level Labor Demand and Factor Price

The market-size effect on factor prices emerges when sector-level labor demand faces the local labor market clearing condition. Aggregating eq. (10) over sectors with $\sum_{k\in\mathcal{K}} x_{nk} = \sum_{k\in\mathcal{K}} s_{nk} = 1$ yields the "wage equation" of this model as

$$\frac{(w_n/\lambda_n)^\sigma}{\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma} = \frac{\left(\sum_{k\in K}\mu_{nk}s_{nk}\right)L_n\lambda_n(w_n/\lambda_n)}{\sum_{i\in\mathcal{N}}\left(\sum_{k\in K}\mu_{ik}s_{ik}\right)L_i\lambda_i(w_i/\lambda_i)}. \tag{11}$$

[8]The absence of $(1-\phi_k)$ and the presence of $(1+\mu_{nk})$ in eq. (10) imply that the second additional effect offsets the diminishing incentive to locate in larger markets. I analyze this mechanism in more detail in **?**.

This equation shows that a factor price $w_n/\lambda_n$ increases with population $L_n$, productivity $\lambda_n$, and expenditure-weighted mean of home-market multipliers $\sum_{k\in\mathcal{K}} \mu_{nk} s_{nk}$ in equilibrium, reflecting the market-size effect on factor prices of this model. To fully analyze the market-size effect, I need one more step because $\mu_{nk}$ is a function of $\{w_i/\lambda_i\}_{i\in\mathcal{N}}$. I factorize the effect into that of overall market size $L_n\lambda_n$—a well-known effect—and that of expenditure composition $\{s_{nk}\}_{k\in\mathcal{K}}$—a new effect. Proposition 1 states the former.

**Proposition 1** (Overall-Market-Size Effect on Factor Prices)**.** *Given expenditure compositions in two cities $\{s_{nk},\ s_{ik}\}_{k\in\mathcal{K}}$, the relative factor price $\frac{w_n}{\lambda_n}/\frac{w_i}{\lambda_i}$ increases with the relative population $L_n/L_i$ and the relative productivity $\lambda_n/\lambda_i$.*

A greater population expands sectoral market size, requiring more entrants or a higher factor price as I analyzed with eq. (7). However, at the city level, entrants do not appear because city-level labor supply is fixed; thus, the factor price must rise, making the city-level labor demand curve upward-sloping. Similarly, a higher productivity, given an income (wage) per efficiency unit, increases expenditures per worker $E_n$; consequently, home sectoral markets grow in value, requiring a higher factor price.

Separately, market composition affects factor prices through the city-level home-market multiplier $\sum_{k\in\mathcal{K}} \mu_{nk} s_{nk}$. I can obtain Theorem 1 from eq. (11) and that $\partial\mu_{nk}/\partial(w_n/\lambda_n) < 0$.

**Theorem 1** (Sectoral-Market-Composition Effect on Factor Prices)**.** *Order sectors according to their tradabilities such that $\phi_k$ increases with $k$. Let high-tradability sectors account for greater shares in an expenditure composition $\{s^a(k)\}_{k\in\mathcal{K}}$ than in $\{s^b(k)\}_{k\in\mathcal{K}}$ in the sense that the cumulative sum of expenditure shares is smaller* for all $k$ (*i.e.,* $\forall k \in \mathcal{K},\ \sum_{x=1}^{k} s^a(x) \le \sum_{x=1}^{k} s^b(x)$)*. Then, a city with $\{s^a(k)\}_{k\in\mathcal{K}}$ commands a higher factor price ($w_n/\lambda_n$) than an equal-sized and equally productive city with $\{s^b(k)\}_{k\in\mathcal{K}}$.*

Theorem 1 coupled with Proposition 1 refine the market-size effect on factor prices, considering the sector-specific power of market size. Labor demand is more responsive to expenditures in high-tradability sectors. Consequently, when a city modifies its demand towards these sectors, it generates greater aggregate labor demand, resulting in a higher factor price. This subsection's analysis is agnostic on mechanisms behind city-specific market compositions. The next subsection shows what determines market compositions in my model.

**Market Composition**

I solve for an expenditure share after substituting eq. (3) for $P_{nk}$ in eq. (7) with $P_n = E_n/C_n$ as

$$s_{nk} = \left\{ \frac{1}{(1-\phi_k)(1+\mu_{nk})} \left( \tilde{\beta}_k^{\frac{1}{1-\eta}} C_n^{\frac{\epsilon(k)}{1-\eta}} \cdot \frac{C_n}{\lambda_n (\lambda_n L_n)^{\frac{1}{\sigma-1}}} \right)^{\sigma-1} \right\}^{\frac{1-\eta}{\sigma-\eta}}. \tag{12}$$

This equation provides two observations. First, a general equilibrium force attenuates and amplifies expenditure patterns when sectors are gross complements ($\eta < 1$) and gross substitutes ($\eta > 1$), respectively. [9] To see this, recall that $\tilde{\beta}_k C_n^{\epsilon(k)}$ is the weight of sector $k$ in city $n$'s expenditure as eq. (3) shows. The product of $\tilde{\beta}_k C_n^{\epsilon(k)}$ and $(P_{nk}/P_n)^{1-\eta}$ becomes the expenditure share, and the logarithmic derivative of which with respect to log $\tilde{\beta}_k C_n^{\epsilon(k)}$ is one. In contrast, that of an expenditure share in eq. (12) is

$$\frac{\partial \log s_{nk}}{\partial \log \tilde{\beta}_k C_n^{\epsilon(k)}} = \frac{\sigma-1}{\sigma-\eta} \begin{cases} > 1 & if\ \eta > 1, \\ < 1 & if\ \eta < 1. \end{cases}$$

A high value of $\tilde{\beta}_k C_n^{\epsilon(k)}$ generates a high expenditure share for that sector, attracting firms and lowering the sectoral price index relative to other sectors.[10] This relative price change attenuates the high expenditure share by substitution when $\eta < 1$. In contrast, it amplifies the expenditure share when $\eta > 1$. The same observation holds when I explicitly focus on the effect of heterogeneous income elasticities as $\partial^2 \log s_{nk}/\partial \log C_n \partial \epsilon(k) = (\sigma-1)/(\sigma-\eta)$. A city with high real income spends relatively more on income-elastic sectors, and whether it is amplified depends on $\eta$.

Second, factor prices influence cities' expenditure shares along the tradability dimension. The logarith-

[9]Matsuyama (2019) discusses this effect in detail.

[10]This demand-driven change in price indices can be interpreted as the Schmookler effect (Schmookler (1966)). Relabeling varieties as intermediate inputs and consumption composites as final products like Matsuyama (2019), a low price index corresponds to high sectoral productivity in final good production.

mic cross derivative of eq. (12) with respect to factor price and tradability is

$$\frac{\partial^2 \log s_{nk}}{\partial(w_n/\lambda_n)\partial\phi_k} \begin{cases} < 0 & if\ \eta > 1, \\ > 0 & if\ \eta < 1, \end{cases} \tag{13}$$

because $\partial^2 \log(1+\mu_{nk})/\partial(w_n/\lambda_n)\partial\phi_k < 0$. A high factor price tilts expenditures towards sectors with high and low tradabilities when $\eta < 1$ and $\eta > 1$, respectively. This force originates from the zero profit condition. A disadvantage in factor price requires mild competition to offset in equilibrium. In other words, firms exit from the sectoral market. Higher tradability aggravates this disadvantage, accelerating firms' exit and bidding up the sectoral price index.[11] The outcome depends on the inter-sectoral price elasticity $\eta$.

### City-Level Labor Demand with Inter-Sectoral Substitution

Result (13) enables us to characterize the relationship between the city size and the factor price—the city-level labor demand curve—considering endogenous expenditure shifts across the tradability spectrum. Suppose all sectors have the same income elasticity ($\forall k$, $\epsilon(k) = \epsilon$). Then, city size increases the factor price, following eq. (11), which in turn shifts expenditures towards sectors with high and low tradabilities when sectors are gross complements and gross substitutes, respectively, following result (13). Theorem 1 suggests that this shift of expenditures amplifies and dampens the rise of the factor price, respectively; clearly, it steepens the upward-sloping labor demand curve with a factor price at the vertical axis when sectors are gross complements (Figure 2). When sectors are gross substitutes, it flattens the curve and, possibly, the slope becomes downward with strong inter-sectoral substitutions (Figure 2).

[11] $\partial^2 \log(P_{nk}/w_n)/\partial w_n \partial\phi_k > 0$, following from eq. (21) in Appendix B.3.

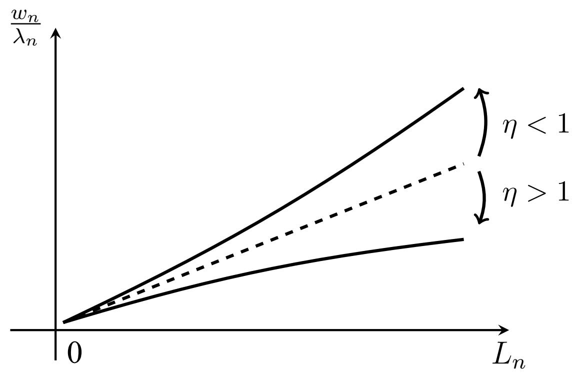



Figure 2: Market-Size Effect on Factor Prices with Inter-Sectoral Substitution

### City-Level Labor Supply

This subsection obtains an equation that expresses city-level labor supply. Plugging the sector-level good demand function (2) into the definition of real income, expressing the price indices using employment shares and city size, and substituting $(a_n/a_1)(L_n/L)_1^\gamma C_1$ for $C_n$ following eq. (4) yield

$$1 = \sum_{k\in\mathcal{K}} \tilde{\beta}_k \left(\frac{a_n}{a_1}\right)^{(1-\eta)(\frac{\epsilon(k)}{1-\eta}+1)} \cdot \Bigg[\lambda_n^{\sigma-1}\Bigg\{(1-\phi_k)x_{nk}L_1\left(\frac{L_n}{L_1}\right)^{1-(\sigma-1)(\frac{\epsilon(k)}{1-\eta}+1)\gamma} + \phi_k\left(\frac{w_n}{\lambda_n}\right)^{\sigma-1}\left(\frac{L_n}{L_1}\right)^{-(\sigma-1)\gamma(\frac{\epsilon(k)}{1-\eta}+1)}\sum_{i\in\mathcal{N}} x_{ik}L_i\left(\frac{w_i}{\lambda_i}\right)^{1-\sigma}\Bigg\}\Bigg]^{\frac{1-\eta}{1-\sigma}}, \tag{14}$$

which uses city 1 as a base. To interpret this equation, I impose Assumption 1.

**Assumption 1.** $\gamma > \frac{1}{\sigma-1}\max_{k\in\mathcal{K}}\theta(k)$ *where* $\theta(k) = [1+\epsilon(k)/(1-\eta)]^{-1}$.

Assumption 1 ensures that the dispersion force from the heterogeneous inherent preference for cities is stronger than the agglomeration force, preventing a city from attracting all workers.[12][13] The factor $1/(\sigma-1)$ is the elasticity of the agglomeration economy or the positive externality in Krugman-type models with homothetic preference; the mass of varieties in a location increases with market size, and consumers have a

[12]Without the term $\max_{k\in\mathcal{K}}\theta(k)$, this condition corresponds to that for the existence of a unique equilibrium of Redding (2016) without residential land use.

[13]It does not contradict Assumption 4. When $\eta > 1$, these assumptions can be rewritten as $\forall k$, $(\sigma-1)^{-1} < \gamma\theta(k)^{-1} < (\eta-1)^{-1}$ . Given the assumption of $\eta < \sigma$, this can be simultaneously satisfied.

love-of-variety preference.[14] The additional factor $\max_{k\in\mathcal{K}} \theta(k)$ reflects that $C_n$ has endogenously varying returns to scale in terms of $\{Q_{nk}\}_{k\in\mathcal{K}}$. The product of these factors is the upper limit of the positive externality from a greater population.

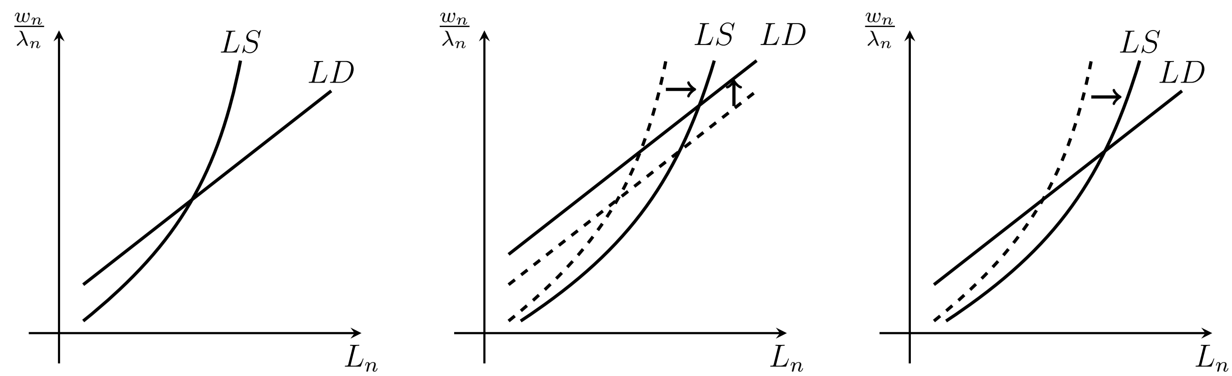
Given Assumption 1 and common terms among cities ($a_1$, $L_1$, and $\sum_{i\in\mathcal{N}} x_{ik} L_i (w_i/\lambda_i)^{1-\sigma}$), eq. (14) shows that a city-level labor supply $L_n$ increases with the factor price $w_n/\lambda_n$ conditional on a level of amenity $a_n$, a productivity $\lambda_n$, and employment composition $\{x_{nk}\}_{k\in\mathcal{K}}$. A high factor price raises purchasing power for varieties produced in other cities, elevating real income. A caveat is that this observation ignores endogenous changes in cities' employment compositions. Nevertheless, it enables us to obtain intuitions for general equilibrium results by the partial equilibrium analysis in the next section.

# 4 Partial Equilibrium Analysis

Given the two equilibrium relationships between city size and the factor price, this section briefly illustrates how cities' productivities and amenities shape patterns of city size and factor prices. Figure 3a depicts the labor-demand curve (LD curve) from the wage equation (11) and the labor-supply curve (LS curve) from eq. (14), holding expenditure and employment compositions constant and taking the common terms as given. [15]

[14] Another interpretation is a positive externality on productivity, as adopted by Matsuyama (2019). When labeling varieties as intermediate inputs and consumption composites as final products, the agglomeration economy works on the productivity of final products.

[15] How the curves intersect is not easily identifiable from the equations in this part. The depiction here is based on the theoretical results in the general equilibrium. The LS curve intersecting the LD curve from below is consistent with the cross-city analysis in the next section. Moreover, it suggests the stability of an equilibrium.




(a) Original State (b) Higher Productivity ($\lambda_n \uparrow$) (c) Better Amenities ($a_n \uparrow$)

Figure 3: Partial Equilibrium Analysis

When city $n$ becomes more productive ($\lambda_n \uparrow$), the LD curve shifts up, and the LS curve shifts to the right, as in Figure 3b. The LD curve shifts because higher productivity implies higher income for a given factor price. Consequently, the home market expands in value, and the factor price must rise to maintain the zero-profit condition. The LS curve shift reflects that real income increases for two reasons. First, higher income improves purchasing power for varieties produced in other cities. Second, higher productivity increases the mass of locally-produced varieties, lowering sectoral price indices. Owing to the shifts of the two curves, the new intersection occurs at higher levels of city size and the factor price compared to before.

When city $n$ has better amenities ($a_n \uparrow$), the LS curve shifts to the right, as in Figure 3c, because the better amenities attract more people. However, the LD curve is unaffected because amenities do not affect production. Consequently, the intersection moves along the LD curve, and the population and factor price are higher than before.[16]

This partial equilibrium analysis predicts that cities with better fundamentals become larger and pay higher factor prices. Although this analysis ignores endogenous changes in expenditure and employment compositions and other cities' populations, these results carry over to general equilibrium as the following sections detail.

[16] Even when the model incorporates inelastic land supply, the same result holds as long as the agglomeration force is stronger than the dispersion force from the inelastic land supply. While this result is contrary to what a Rosen-Roback model (Rosen 1979; Roback 1982) implies, it is not new in the literature. For example, Glaeser and Gottlieb (2009) point out that rising amenities can increase wages because of agglomeration economy.

# 5 Cross-City Analysis with Only Sector-Specific Income Elasticities

This section demonstrates how differences in the fundamentals generate cities' patterns of population, factor prices, expenditures, and employment in general equilibrium, replicating the employment pattern in Figure 1. To this end, I strip the model of heterogeneous tradability by assuming $\phi_k = \phi$ for all $k$ in this section and focus on the effects that heterogeneous income elasticities alone generate. In the next section, I reconsider heterogeneous tradability to explore the potential implications of the demand-side mechanism.

The analysis of equilibrium existence precedes the cross-city analysis. To prove the existence, I adapt the technique used by Zeng and Uchikawa (2014), who study a model of many equidistant countries. They construct a map of wages and apply Brouwer's fixed-point theorem to prove the existence. Extending their method to my model requires identifying the city with the highest factor price in equilibrium. An additional weak assumption makes this identification feasible, which I impose as Assumption 2 after defining an income fundamental.

**Definition 1** (Income Fundamental)**.** City $n$ has a better income fundamental than city $i$ if only if $a_n \lambda_n^{\gamma+\theta(k)} > a_i \lambda_i^{\gamma+\theta(k)}$ for all $k$.

**Assumption 2.** *There exists a city that has a better income fundamental than any other city.*

In the rest of this section, I designate the city with the best income fundamental as city 1, setting $w_1 = \lambda_1 = a_1 = 1$ without loss of generality. Subsequently, Assumption 2 ensures that city 1 has the highest factor price in equilibrium, although I discuss the order of factor prices later in detail.[17] Assumption 2, in addition to the dispersion force being more potent than the agglomeration force (Assumption 1), suffices to show the existence of an equilibrium as Lemma 1.

**Lemma 1** (Existence of Equilibrium)**.** *Given Assumptions 1 and 2 and common tradability, there exists an equilibrium.*

[17] The inequality in Definition 1 is strict because it simplifies the exposition. A weak inequality suffices for the existence of an equilibrium.

The endogenous expenditure composition coupled with more than two cities makes it infeasible to prove the uniqueness and stability generally.[18] Nevertheless, all the following cross-city comparisons hold in all the possible equilibria in which all sectors have non-zero production in all the cities. Further, Appendix C proves that the equilibrium is stable and unique for the case of two cities. The rest of this section compares cities in an equilibrium with Assumptions 1 and 2 and a common trade cost.

### City Size and Factor Prices

The first cross-city comparison concerns city size. The partial equilibrium analysis (Figures 3b and 3c) suggests that cities with better fundamentals become larger. This result remains the same, and Proposition 2 considers the two fundamentals jointly.

**Proposition 2** (Pattern of City Size)**.** *Suppose that a city has higher values of* $a_n\lambda_n^{(1+(\sigma-1)^{-1})\theta(k)}$ *and* $a_n\lambda_n^{\theta(k)}$ *than another city for all* $k$*. Then, the city has a greater population.*

The condition in Proposition 2 ensures that city $n$ offers a higher utility before considering the Fréchet utility shock ($C_n a_n > C_i a_i$). A higher productivity generates the agglomeration economy by increasing the mass of varieties that given labor produces. As discussed with Assumption 1, $(\sigma-1)^{-1}\theta(k)$ measures the strength of the agglomeration economy in this model. However, the condition on $a_n\lambda_n^{(1+(\sigma-1)^{-1})\theta(k)}$ does not suffice in Proposition 2 for two reasons. First, a higher productivity contributes to other cities' agglomeration economies by trade though it does so with attritions. Second, residents consume imports, the prices of which local productivity does not directly affect. In sum, inter-city trade attenuates cross-city variations in the agglomeration economy, lowering the elasticity of a relative agglomeration economy with respect to relative productivity. The condition of Proposition 2 ensures that the aggregate effect from a relative amenity level and relative productivity is greater in city $n$, regardless of the degree of the attenuation.

The second cross-city comparison concerns factor prices. The result of the partial equilibrium analysis remains the same again as Proposition 3.

**Proposition 3** (Pattern of Factor Prices)**.** *A city with a better income fundamental commands a higher factor price.*

[18] Cities are not atomic in this model, so the proof of stability requires the analysis of all possible cases of migrations.

Assumption 1 implies $(1 + (\sigma - 1)^{-1})\theta(k) < \gamma + \theta(k)$; thus, relative productivity weighs more in the factor-price pattern (Proposition 3) than in the city-size pattern (Proposition 2). This result reflects that the home-market effect couples city size with a productivity to determine a factor price (Proposition 1). A higher value of $\gamma$ makes productivity have greater weights in the order of income fundamentals; a strong dispersion force weakens the relationship between amenities and city size, making productivities relatively more influential in wage determination.

Thus, generally, cities with better fundamentals—productivities and amenities—become larger and command a higher factor price. Particularly, when I condition one of the fundamentals, the order of city size perfectly coincides with that of nominal income. Recall nominal income $E_n$ equals $w_n$ in this model.

**Corollary 1** (City-Size and Nominal-Income Patterns Conditional on One Fundamental)**.** *A city with a better fundamental conditional on the other becomes larger and the residents earn higher nominal income.*

Workers in large cities tend to have higher nominal income in equilibrium. This result is consistent with the stylized fact that nominal income is higher in larger cities, even when we control for observable or unobservable workers' characteristics (Behrens and Robert-Nicoud 2015; Glaeser and Mare 2001).

### Expenditure, Trade, and Employment

A city becomes large by offering a high utility before considering the Fréchet utility shock ($C_n a_n > C_i a_i$). Subsequently, a cross-city expenditure pattern is clear when we condition on cities' amenity level. Residents in larger cities spend more on the income-elastic sectors, as we saw $\partial^2 \log s_{nk} / \partial \log C_n \partial \epsilon(k) = (\sigma - 1)/(\sigma - \eta) > 0$ in Section 3. City-specific amenity levels possibly complicate this expenditure pattern. Nevertheless, I can obtain Proposition 4.

**Proposition 4** (Expenditure Pattern)**.** *A sectoral expenditure share ratio of a larger city to a smaller city conditional on one fundamental is greater in a more income-elastic sector; that is,* $\forall (n,i) \in \{(n,i) \in \mathcal{N}^2 |\ L_n > L_i \wedge a_n = a_i\} \cup \{(n,i) \in \mathcal{N}^2 |\ L_n > L_i \wedge \lambda_n = \lambda_i\}$, $\forall (k,\ell) \in \{(k,\ell) \in \mathcal{K}^2 | \epsilon(k) > \epsilon(\ell)\}$, $s_{nk}/s_{ik} > s_{n\ell}/s_{i\ell}$.

Thus, residents in larger cities spend more on income-elastic sectors even when conditioned on cities' productivity. This result reflects that better amenities attract workers, generating the agglomeration economy.

The home-market effect amplifies these expenditure patterns into trade patterns as Lemma 2, reflecting the force in eq. (10).

**Lemma 2** (The Home-Market Effect)**.** *There is a relationship between expenditure shares and employment shares such that*

$$x_{nk} = s_{nk} + \mu_n (s_{nk} - s_k), \tag{15}$$

*where*

$$\mu_n = \left[ \frac{(\phi^{-1} + N - 1)(w_n/\lambda_n)^\sigma}{\sum_{j \in \mathcal{N}} (w_j/\lambda_j)^\sigma} - 1 \right]^{-1} > 0, \quad s_k = \sum_{i \in \mathcal{N}} \frac{L_i w_i \mu_i}{\sum_{j \in \mathcal{N}} L_j w_j \mu_j} s_{ik}.$$

*Accordingly, a city is a net exporter in each sector with expenditure share $s_{nk}$ higher than the nationwide weighted mean $s_k$.*

Employment shares increase more than one-for-one with expenditure shares. Figure 4 depicts eq. (15).

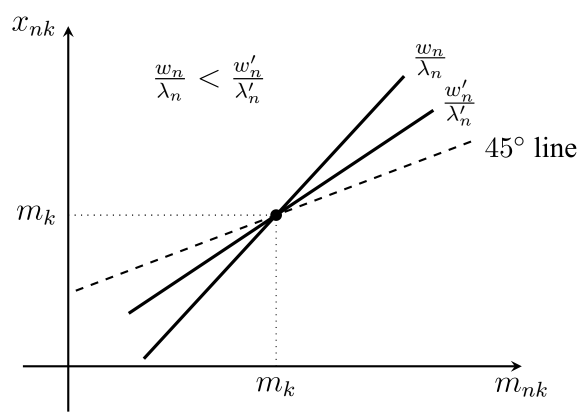



Figure 4: Home-Market Effect on Trade with Common Trade Cost

The slope—one plus a home-market multiplier $\mu_n$—depends on the factor price ($w_n/\lambda_n$), but it always intersects the 45-degree line from below at the point where the expenditure share equals the nationwide level $s_k$. When an expenditure share is greater than $s_k$, city $n$ becomes a net exporter.[19] A higher factor price attenuates this amplification as the slope becomes flatter; in other words, a higher factor price erodes the

[19]Net exports increasing with demand corresponds to the "strong home-market effect," defined by Costinot et al. (2019).

competitiveness of local firms, diminishing the power of a large market. This result implies that large cities tend to have less production specialization than small cities, which is consistent with the stylized fact that large cities have more industrial diversification (Duranton and Puga (2000)).[20] Corollary 2 describes cities' exporter-importer statuses, according to Lemma 2 and the expenditure pattern of Proposition 4.

**Corollary 2** (Trade Pattern with Heterogeneous Income Elasticities)**.** *Suppose that there exist two cities that share the level of either productivity or amenities, and the large one is a net exporter while the small one is a net importer in a sector. Then, in a more income-elastic sector, the larger one is a net importer only if the smaller one is a net importer, and the smaller one is a net exporter only if the larger one is a net exporter.*

In short, large and small cities tend to be net exporters in income-elastic and income-inelastic sectors, respectively, because they tend to have expenditure shares higher than the nationwide weighted mean for corresponding sectors. This is an $N$-equidistant-city version of the home-market effect on trade patterns. Variations in relative market size generate comparative advantages, translating into the employment pattern with amplification. Unlike Krugman (1980), who assumes an exogenous preference difference to generate the heterogeneous relative demand, and similar to Matsuyama (2019), differences in relative market size arise endogenously from the non-homothetic preference.

Cities' production pattern replicates Figure 1. Consider two cities according to eq. (15) or Figure 4 by division into cases. First, suppose one of cities $n$ and $i$ is a net exporter, and the other is a net importer. Then, $s_{nk} > s_{ik}$ implies $x_{nk} > x_{ik}$. Second, suppose both cities are net exporters. Then, $s_{nk} > s_{ik}$ and $w_n/\lambda_n < w_i/\lambda_i$ imply $x_{nk} > x_{ik}$. Finally, suppose both cities are net importers. Then, $s_{nk} < s_{ik}$ and $w_n/\lambda_n < w_i/\lambda_i$ imply $x_{nk} < x_{ik}$. In equilibrium, small cities tend to have lower $w_n/\lambda_n$, and they tend to have higher and lower $s_{nk}$ in income-inelastic and income-elastic sectors, respectively, following Corollary 1 and Proposition 4. Thus, small cities tend to have relatively large employment (high $x_{nk}$) in income-inelastic sectors, replicating Figure 1.[21]

[20] In two-location models, goods market clearing conditions imply less specialization in the larger location because one country's trade surplus equals the other's trade deficit. Unlike those models, goods market clearing conditions do not, at least not directly, imply this result in this many-location model.

[21] This analysis left out two cases: one where both cities are net exporters and $s_{nk} > s_{ik}$ and $w_n/\lambda_n > w_i/\lambda_i$, and another where both cities are net importers and $s_{nk} < s_{ik}$ and $w_n/\lambda_n > w_i/\lambda_i$. The weaker amplification from the

# 6 Cross-City Analysis with Added Sector-Specific Trade Costs

This section explores potential implications of the demand-side mechanism using the model with sector-specific income elasticities and trade costs. The additional heterogeneity makes the model more complex. Owing to this complexity, I assume that cities differ only in productivities to keep the analysis simple, and I let city 1, without loss of generality, be the fundamentally most productive city.[22]

**Assumption 3.** $\forall n\ a_n = 1$, *and* $\forall n \geq 2\ \lambda_n < \lambda_1$.

Additionally, I impose Assumption 4.

**Assumption 4.** $\eta < \max\{1, 1 + \min_{k \in \mathcal{K}} \epsilon(k) + 1/\gamma\}$.

As discussed in Section 3, inter-sectoral substitution with $\eta > 1$ can attenuate the market-size effect on factor prices through market composition. Assumption 4 ensures the positive slope of city-level labor demand, which is crucial in obtaining analytical results. Assumption 4 is reasonable to impose when the number of sectors is not too large because Comin et al. (2021) estimated the inter-sectoral price elasticity to be 0.07–0.13 using ten sectors. Subsequently, one can prove the existence of an equilibrium as Lemma 3.

**Lemma 3** (Existence of Equilibrium)**.** *Given Assumptions 1, 3, and 4, there exists an equilibrium.*

All the following cross-city comparisons hold in all the possible equilibria in which all sectors have non-zero production in all the cities. For the remainder of this section, we compare cities in an equilibrium, under Assumptions 1, 3, and 4.[23]

### City Size and Factor Prices

Similar to Corollary 1 for the common tradability case, the order of productivities becomes that of factor prices and, consequently, that of nominal income as Lemma 4.

higher factor price makes it infeasible to tell which city has a greater employment share. Two-location models do not have this issue because the two locations cannot simultaneously be net exporters or importers.

[22] I assume that city 1 is strictly more productive than any other city to make the following exposition clear, but the strictness is by no means necessary.

[23] The analysis of the uniqueness and stability is left for future research. An earlier version of this paper assumes that sectors are either tradable or untradable and shows that the equilibrium is unique and stable for the two-city case.

**Lemma 4** (Patterns of Factor Prices and Nominal Income). *A fundamentally more productive city commands a higher factor price, and the residents earn higher nominal income.*

The order of city size, unlike factor prices, is generally infeasible to pin down because heterogeneous tradabilities provide cities with a way to raise real income, not relying on good fundamentals—specialization in low-tradability sectors. To understand this possibility, suppose some sectors are extremely tradable, and the others are hardly tradable. Then, the price indices of the extremely tradable sectors do not differ much across cities, whereas the price indices of the hardly tradable sectors are relatively inexpensive in cities that produce rich varieties in these sectors; therefore, cities specializing in the hardly tradable sectors enjoy the inexpensive price indices while not inflating the price indices of the extremely tradable sectors, raising real income.[24] Assumption 5 weakens this mechanism and eliminates the possibility that any other city becomes the largest as Lemma 5.

**Assumption 5.** $(\bar{\phi}^{-1} - 1)/(\underline{\phi}^{-1} - \bar{\phi}^{-1}) \geq (1 - \lambda_n^{\eta})/(\lambda_n^{\eta} - \lambda_n^{\sigma})$ *for all* $n \geq 2$ *where* $\bar{\phi} = \max_{k \in \mathcal{K}} \phi_k$ *and* $\underline{\phi} = \min_{k \in \mathcal{K}} \phi_k$.

**Lemma 5** (Largest City). *Given Assumption 5 in addition, the most fundamentally productive city becomes the largest city.*

The more modestly tradabilities differ (i.e., the smaller $\underline{\phi}^{-1} - \bar{\phi}^{-1}$ is) and the lower tradabilities generally are (i.e., the greater $\bar{\phi}^{-1} - 1$ is), the more likely Assumption 5 holds. Separately, a larger intra-sectoral price elasticity $\sigma$ and a smaller inter-sectoral price elasticity $\eta$ makes it more likely. The former makes a price index insensitive to the mass of variety, dampening the effect of specialization. The latter prevents inter-sectoral substitutions from facilitating the supply-side specialization in low-tradability sectors. Consequently, the largest city has the highest factor prices as Corollary 3.

**Corollary 3.** *Given Assumption 5 in addition, the largest city commands the highest factor price, and the residents earn the highest nominal income.*

[24]This analysis holds the income level constant. Specializing in low-tradability sectors has another force: it reduces nominal income (Theorem 1).

### Expenditure, Trade, and Employment

The model has two expenditure patterns. The first one is the same as Section 5 except that I need to condition on tradability in this case. The sectoral expenditure share ratio of the larger city to the smaller city is greater in a more income-elastic sector conditional on tradability.[25] Thus, the force from cross-city real income inequality is still operative. Given Theorem 1, it suggests that cross-city nominal income inequality widens when high-tradability sectors have high income elasticities.

Heterogeneous trade costs generate the second expenditure pattern. Inequalities (13) showed that a high factor price tilts expenditures towards sectors with high and low tradability when $\eta < 1$ and $\eta > 1$, respectively. I can isolate this force in equilibrium by conditioning income elasticity as Proposition 5.

**Proposition 5** (Expenditure Pattern across Tradability Spectrum)**.** *The sectoral expenditure share ratio of a higher-factor-price city to a lower-factor-price city is greater in a sector with a higher and lower tradability conditional on income elasticity when sectors are gross complements and substitutes, respectively; that is,* $\forall (n,i) \in \{(n,i) \in \mathcal{N}^2 \mid w_n/\lambda_n > w_i/\lambda_i\}$, $\forall (k,\ell) \in \{(k,\ell) \in \mathcal{K}^2 \mid \phi_k > \phi_\ell,\ \epsilon(k) = \epsilon(\ell)\}$, $s_{nk}/s_{ik} > s_{n\ell}/s_{i\ell}$ *for* $\eta < 1$*, and* $s_{nk}/s_{ik} < s_{n\ell}/s_{i\ell}$ *for* $\eta > 1$.

*Remark.* Given Lemma 4, a higher factor price implies a higher wage and income.

Recall that the largest city commands the highest factor price when Assumption 5 holds. Given Theorem 1, this result suggests that endogenous expenditure composition amplifies and dampens a city-size wage premium ($dw_n/dL_n$) by the market-composition effect when sectors are gross complements and gross substitutes, respectively.

Trade patterns reflect sector-specific amplification strengths as well as relative market size. In the common-tradability case, expenditure shares determine the net exporter-importer status. Heterogeneous tradability makes it *weighted relative market size*—a product of $s_{nk}$ and $\mu_{nk}$, divided by the sum of products—as stated in Lemma 6 and illustrated in Figure 5.

[25] To verify this result, notice that a common value of $\phi_k$ implies the same value for the factor $(1-\phi_k)(1+\mu_{nk})$ in expenditure shares of eq. (12).

**Lemma 6** (Home-Market Effect with Sector-Specific Trade Costs)**.** *Given an equilibrium, there is a relationship between expenditure shares and employment shares such that*

$$\tilde{x}_{nk} = \tilde{s}_{nk} + \mu_{nk}(\tilde{s}_{nk} - \tilde{s}_k), \tag{16}$$

*where*

$$\tilde{x}_{nk} = \frac{\mu_{nk} x_{nk}}{\sum_{\ell \in \mathcal{K}} \mu_{n\ell} s_{n\ell}}, \quad \tilde{s}_{nk} = \frac{\mu_{nk} s_{nk}}{\sum_{\ell \in \mathcal{K}} \mu_{n\ell} x_{n\ell}}, \quad \tilde{s}_k = \sum_{i \in \mathcal{N}} \frac{L_i w_i \sum_{\ell \in \mathcal{K}} \mu_{i\ell} s_{i\ell}}{\sum_{j \in \mathcal{N}} L_j w_j \sum_{\ell \in \mathcal{K}} \mu_{j\ell} s_{j\ell}} \tilde{s}_{ik}.$$

*Accordingly, a city is a net exporter in each sector whose weighted relative market size $\tilde{s}_{nk}$ is higher than the corresponding nationwide weighted mean $\tilde{s}_k$ ($x_{nk} > s_{nk} \iff \tilde{s}_{nk} > \tilde{s}_k$).*

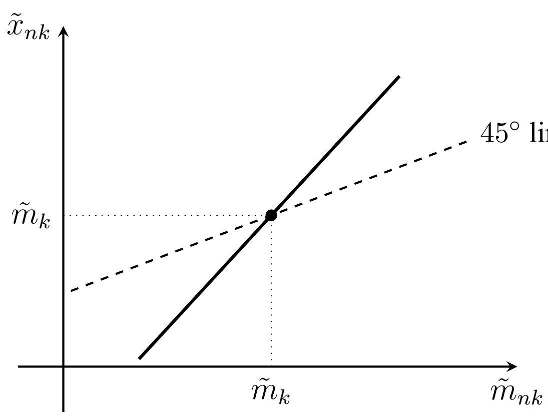



Figure 5: Home-Market Effect with Sector-Specific Trade Costs

Cities have comparative advantages in sectors with high $\tilde{s}_{nk}$ in the way analogous to the common-tradability case (Lemma 2). The formula of $\tilde{s}_{nk}$ shows the importance of tradabilities of other sectors for which the city has relatively large markets. For example, consider sectors $k$ and $\ell$ whose relative market sizes $s_{nk}$ and $s_{n\ell}$ are equally large in city $n$, and recall that $\mu_{nk}$ increases with $\phi_k$. When sector $k$ has a high tradability, $\tilde{s}_{n\ell}$ becomes low, making city $n$, ceteris paribus, less likely to be a net exporter in sector $\ell$, and vice versa. Thus, the tradability of sector $k$ affects the net-exporter-importer status in sector $\ell$. This result is intuitive because high tradability reinforces the power of market size, drawing more workers into sector $k$. Consequently, the factor price rises, making firms in sector $\ell$ less competitive in the nationwide market.

Nevertheless, standard relative market size is still influential. Sectors with the same tradability have the same home-market multiplier in a city, implying $\tilde{s}_{nk}/\tilde{s}_{n\ell} = s_{nk}/s_{n\ell}$; thus, similar to Proposition 4, larger cities have comparative advantages in income-elastic sectors, conditional on tradability: $\forall(k,\ell) \in \{(k,\ell) \in \mathcal{K}^2 | \epsilon(k) > \epsilon(\ell), \phi_k = \phi_\ell\}$, $\tilde{s}_{nk}/\tilde{s}_{ik} > \tilde{s}_{n\ell}/\tilde{s}_{i\ell}$. [26]

### Discussion on Manufacturing-Services Economy

To obtain concrete implications of the market-composition effect, I consider a specific example: an economy of manufacturing and services. Services are characterized by higher income elasticity and lower tradability relative to manufacturing (Comin et al. (2021), Anderson et al. (2014)).[27] These characteristics suggest that a city-size wage premium possibly attenuates, owing to the sectoral market-composition effect. Two forces influence cities' expenditure composition. On the one hand, high real income in large cities drives expenditures toward services. On the other hand, large cities tend to feature high factor prices, which is a disadvantage especially for manufacturing because of its high tradability, raising their sectoral price index. Consequently, as Proposition 5 shows, expenditures shift toward manufacturing when $\eta < 1$. However, when $\eta$ is close to one, the second force becomes negligible. These considerations suggest that the net effect makes large cities spend more on services, which have low tradability, leading to a smaller city-size wage premium ($dw_n/dL_n$).

Conversely, this analysis suggests that an improvement in services' tradability potentially raises a city-

[26]Another force makes cities with low and high factor prices tend to be net exporters in high- and low-tradability sectors, respectively. I refer the readers to **?** for a detailed discussion of this result. A short explanation is that, as Section 3 showed, high tradability, ceteris paribus, makes relative market size more powerful in attracting firms captured by that $\partial\mu_{nk}/\partial\phi_k > 0$. An inexpensive factor price reinforces this effect as $\mu_{nk}$ is log-supermodular in $(w_n/\lambda_n)^{-1}$ and $\phi_k$ ($\partial^2 \log \mu_{nk}/\partial(w_n/\lambda_n)^{-1}\partial\phi_k > 0$). Thus, high-tradability sectors have even higher $\mu_{nk}$ in cities with lower factor prices. This pattern of $\mu_{nk}$ dominates the pattern of relative market size $s_{nk}$ across the tradability spectrum in determining $\tilde{s}_{nk}$, regardless of the level of $\eta$. Hence, the pattern of net exports can be opposite to that of expenditure shares. This result highlights the importance of exploiting exogenous demand factors, instead of relying on expenditure shares, in tests of the home-market effect, similar to Costinot et al. (2019).

[27]Using Canada's provincial data from 1997 to 2007, Anderson et al. (2014) estimate that geography reduces services trade some seven times more than goods trade overall.

size wage premium. Tradability is not constant over time, and trade costs in services have steadily fallen since 1980 because of the fall of communication costs (Head et al. (2009); Eckert (2019)). When tradabilities become uniform across sectors, expenditure compositions no longer affect wages, nullifying the attenuation effect on a city-size wage premium.[28]

# 7 Robustness Checks

In this section, I provide robustness checks that address two supply-side alternative explanations of cities' specialization pattern in Figure 1. The first one is skilled-labor supply in cities. Skilled workers, simultaneously high-income earners, tend to reside in large cities, and those cities tend to host skill-intensive sectors (Davis and Dingel 2020). The second one is sector-specific strengths of agglomeration externalities; firms in sectors that benefit more from agglomeration economy locate in larger cities (Gaubert (2018)). These supply-side mechanisms can generate the employment pattern in Figure 1 if sectors' income elasticities correlate with their supply-side characteristics. Indeed, as is well known, there is a positive correlation between skill intensities and income elasticities (Caron et al. 2014, 2020). I address these concerns in two ways. First, I test if the positive relationship in Figure 1 is robust to controlling for the skilled-labor supply in MSAs. Second, I test a model prediction of an employment pattern conditional on city size. I implement regressions for multiple years separately in each test, which serve as additional robustness checks.

## Controlling for Skilled-Labor Supply

The first robustness check modifies the two-step regression analysis that produced Figure 1. The first step estimates the elasticity of employment with respect to MSA's population conditional on skilled-labor supply; the regression model is given by

[28] The insight in this subsection also applies to international income inequality. If anything, the absence of workers' mobility makes the analysis more straightforward. Productive or large countries command high factor prices and high real income, becoming high-income countries and spending more on services—the sector with high income elasticity and low tradability. Services' trade cost reduction can widen income inequality between countries.

$$L_{ik} = \alpha_k \cdot \exp\Bigg( \xi_k^P \log(Population_i) + \gamma_{1k} \log(College_i) + \gamma_{2k} \log(1 + College_i) + \sum_{region} \gamma_{3k} D_{i,k,region} \Bigg) \cdot e_{ik},$$

where $L_{ik}$, $Population_i$, and $College_i$ are the employment of sector $k$, the population, and the college employment ratio, respectively, of MSA $i$, $D_{i,k,region}$ is a region dummy variable for sector $k$ taking a value of one if MSA $i$ belongs to that region$\in$ $\{Northeast,\ Midwest,\ South,\ West\}$, and $e_{ik}$ is the error term. I use the population data from the Bureau of Economic Analysis (BEA) and the employment data from the County Business Patterns (CBP). I calculate the college employment ratios for full-time workers in MSAs from U.S. Census data via Integrated Public Use Microdata Series (IPUMS) (Ruggles et al. 2023). The model includes two terms with $College_i$ to purge the skill-supply effect from estimated conditional population elasticities of employment $\hat{\xi_k^P}$.

As the regression model shows, I implement level regressions by the Poisson Pseudo Maximum Likelihood (PPML) estimation. As discussed in Silva and Tenreyro (2006), log-linear regressions require a considerably specific condition on error terms to obtain consistent estimators. Moreover, it is problematic in log-linear estimations when zeros are in the data. In contrast, PPML provides consistent estimators that do not require this, and it is efficient with various error term patterns. For this reason, PPML is extremely common in gravity equation estimations in international trade where zeros are prevalent and error terms show heteroskedasticity. In my dataset, 9.5% of the sample is zero. To address these zeros and obtain consistent estimators, I use PPML.

The second-step regression estimates the relationship between income elasticities of demand and the estimated conditional population elasticities of employment with the following model.

$$\hat{\epsilon_k} = \alpha + \beta \hat{\xi_k^P} + e_k,$$

where $\hat{\epsilon}_k$ is an estimated income elasticity of demand for sector $k$ output, $\hat{\xi_k^P}$ is the estimate from the first step, and $e_k$ is the error term for sector $k$. I borrow estimates of income elasticities from Caron et al. (2020), who obtained them through a structural estimation with international trade data. Using an estimate as a regressor generally causes an attenuation bias from sampling errors, but $\hat{\xi}_k^P$ has negligible standard errors from the first step, alleviating the concern.[29] This second-step regression weights sectors by their total employment in the sample to prevent small sectors like forestry from influencing the result excessively. Table 1 summarizes the results of this cross-sectional regression for three years.

Table 1: Results with Population Elasticity of Employment

| | Income elasticity of demand | | | | | |
|---|---|---|---|---|---|---|
| | 2006 | 2011 | 2016 | 2006 | 2011 | 2016 |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Pop. elasticity of emp. ($\hat{\beta}$) | 0.64*** | 0.69*** | 0.71*** | 0.41* | 0.63*** | 0.71*** |
| | (0.18) | (0.17) | (0.19) | (0.23) | (0.20) | (0.23) |
| College ratio controls (1st st.) | | | | ✓ | ✓ | ✓ |
| Observations | 29 | 29 | 29 | 29 | 29 | 29 |
| $R^2$ | 0.23 | 0.32 | 0.29 | 0.06 | 0.15 | 0.17 |
| Adjusted $R^2$ | 0.20 | 0.29 | 0.26 | 0.03 | 0.12 | 0.14 |

*Notes:* ***,**, and * denote significance at the 1%, 5%, and 10% levels, respectively. Bootstrap standard errors are in parentheses. This table displays the coefficients of regressing sectors' income elasticities of demand on their elasticities of employment with respect to MSA's population. The first-stage regression, which obtains the elasticities of employment, control for region $\in$ {Northeast, Midwest, South, West} for (1)-(6) and log(college ratio) and log(1 + college ratio) for (4)-(6). Employment data are from Country Business Pattern data, income elasticities are from Caron et al. (2020), and MSAs' population data are from the Bureau of Economic Analysis.

The results show that the positive relationship is robust to the skilled-labor controls. The three left columns contain results without the controls of the college employment ratio, the 2016 result of which I used to create Figure 1. $\hat{\beta}$ is significantly positive in all three years. The right three columns display the results when controlling for the college employment ratio. It reduced the size of $\hat{\beta}$ in 2006, downgrading the significance to the 10% level. However, $\hat{\beta}$ is significantly positive in the other two years at the 1% level.

[29]The median of standard errors of $\hat{\xi}_k^P$ in the first-step regression is 0.0012, while $\hat{\xi}_k^P$ ranges from 0.19 to 1.52; whereas Caron et al. (2020) report that standard errors for their estimates are between 0.05 and 0.2 for most sectors, and $\hat{\epsilon}_k$ in my sample ranges from 0.36 to 1.41.

To gauge the size of the effect, suppose two cities have populations in the first and third quartiles and the same college-worker ratio. The coefficient size of 0.71 in 2016 means that a sector with a 0.1 higher income elasticity boasts a 10 percent higher employment share in the larger city.[30] Thus, the demand-side effect is econometrically and economically significant even after controlling for skilled-labor supply.

## Employment Pattern Conditional on City Size

The theoretical model navigates the second robustness check, which aims to mute the supply-side mechanisms. City size is the common source of comparative advantages in the theories of Davis and Dingel 2020 and Gaubert (2018), generating agglomeration externalities for workers and sectors. To cancel these mechanisms, I focus on a cross-city employment pattern conditional on city size. In my model, given two equal-sized cities, one must have a higher productivity, and the other must have a higher level of amenities, similar to a Rosen-Roback model, unless they have the same levels for both. The one with higher productivity commands higher nominal and real income, making it equally attractive to workers despite the worse amenities. Subsequently, different real income generates different sectoral demand. Formally, I obtain Lemma 7.

**Lemma 7** (Expenditure Pattern Conditional on City Size)**.** *Given Assumptions 1 and 2 and a common trade cost, suppose that two cities are equal-sized in equilibrium. Then, the sectoral expenditure share ratio of the higher-income city to the lower-income city is higher in a more income-elastic sector; that is,* $\forall (n,i,k,\ell) \in \{(n,i,k,\ell) \in \mathcal{N}^2 \times \mathcal{K}^2 \mid L_n = L_i,\ E_n > E_i,\ \epsilon(k) > \epsilon(\ell)\}$, $s_{nk}/s_{ik} > s_{n\ell}/s_{i\ell}$.

Households in the higher-income city enjoy higher real income, spending more on income-elastic sectors. As discussed in Section 5, this expenditure pattern translates into an employment pattern, according to eq. (15), generating a pattern that a high-income city tends to have high employment shares in income-elastic sectors.

To test this prediction, I implement another two-step regression analysis. The first-step regression estimates the income elasticity of employment with respect to MSA's income per capita conditional on MSA's population $\xi_k^I$. The model is given by

[30]This description interprets the result according to the reverse regression formula $\hat{\xi_k^P} = \hat{\beta}^{-1}\alpha + \hat{\beta}^{-1}\hat{\epsilon_k} + \hat{\beta}^{-1}e_k$.

$$L_{ik} = \alpha_k \cdot \exp\left( \xi_k^I \log(Income_i) + \xi_k^P \log(Population_i) + \sum_{region} \gamma_k D_{i,k,region} \right) \cdot e_{ik},$$

where $Income_i$ is per capita personal income of MSA $i$, data of which are from the BEA. The second step regresses income elasticities on these $\hat{\xi}_k^I$ by

$$\hat{\epsilon}_k = \alpha + \beta \hat{\xi}_k^I + e_k.$$

The theoretical model predicts $\beta > 0$.

Table 2: Results with Conditional Income Elasticity of Employment

| | Income elasticity of demand | | |
|---|---|---|---|
| | 2006 | 2011 | 2016 |
| | (1) | (2) | (3) |
| Cond. income elasticity of emp. ($\hat{\beta}$) | 0.134*** | 0.111*** | 0.129*** |
| | (0.038) | (0.039) | (0.043) |
| Observations | 29 | 29 | 29 |
| $R^2$ | 0.316 | 0.229 | 0.246 |
| Adjusted $R^2$ | 0.291 | 0.200 | 0.219 |

*Notes:* ***,**, and * denote significance at the 1%, 5%, and 10% levels, respectively. Bootstrap standard errors are in parentheses. This table displays the coefficients of regressing sectors' income elasticities on their elasticities of employment with respect to MSA's income per capita. The first-stage regression, which obtains the elasticities of employment, control for MSA's population and region $\in$ {Northeast, Midwest, South, West}. Employment data are from Country Business Pattern data, income elasticities are from Caron et al. (2020), and MSAs' population data are from the Bureau of Economic Analysis.

Table 2 shows that the regression results are consistent with this prediction; $\hat{\beta}$ is significantly positive in all three years. Sectors that employ relatively more workers in cities with higher income, conditional on city size, tend to be income-elastic. Thus, the specialization pattern is robust to the alternative explanation by sector-specific strengths of agglomeration externalities. In theory, the effect of skilled-labor supply is also absent in this test. In reality, it survives if skilled workers cluster in some cities for reasons that city size alone cannot explain (e.g., Diamond (2016)). Nevertheless, the two robustness tests jointly demonstrate that the demand-side effect in cities' specialization pattern is robust to the supply-side explanations.

# 8 Conclusion

Beyond explaining cities' specialization patterns from the demand-side perspective, my model implies that endogenous expenditure composition can amplify or attenuate cross-location income inequality. As expenditure patterns substantially vary across cities and countries, using multiple-sector models with sector-specific trade costs can improve quantitative analysis in future research.

I expect supply-side comparative advantages to also affect factor prices through the newfound channel. On the one hand, relatively higher productivity in a high-tradability sector reduces the sectoral price, lowering the expenditure share when sectors are gross complements. On the other hand, I conjecture relative productivity and relative tradability are complementary in generating labor demand. This channel deserves further theoretical and empirical investigation.

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# Appendix

## A Data

To create Figure 1, I borrow estimates of income elasticities from Caron et al. (2020). Using 1997 international trade data for 109 countries, they estimate the elasticities for 49 sectors. The elasticity varies from 0.137 for "Processed rice" to 1.311 for "Financial services nec." I use datasets from County Business Patterns (CBP) for employment data. CBP provides employment data of MSAs annually for sectors classified according to the North American Industry Classification System (NAICS). Caron et al. (2020) use Global Trade Analysis Project (GTAP) sector classification, which differs from NAICS. In most cases, one GTAP sector code corresponds to multiple 3-digit or 4-digit NAICS codes. Following Carrico et al. (2012) and mapping NAICS data to GTAP sectors, I create employment data by GTAP sectors. CBP datasets do not contain employment-size information for MSA-NAICS pairs with fewer than three establishments from 2017. CBP provides employment size class data (e.g., "25,000–49,999") for these pairs before 2017. Small employment values are informative in detecting specialization patterns; therefore, I use employment data up to 2016 throughout the study and the midpoint of the employment size class for pairs only with employment size class information. I drop sectors whose employment is zero in more than 90 percent of MSAs in 2016.

The regression procedure for Figure 1 is the same as the first robustness check in Section 7 except that it does not control for the college employment ratio. It uses PPML and calculates standard errors by a bootstrap procedure that resamples 500 times with replacement over MSAs and sectors separately before the first stage; each sample has a different set of MSAs and a different set of sectors, but all MSAs, in a given sample, have the same set of sectors for employment data. I calculate the standard errors in the robustness checks in the same way.

The first robustness check in Section 7 uses college employment ratios that I measure by college workers to non-college workers working in a given MSA. I use the U.S. census data via IPUMS on 25–55-year-old workers whose "Usual hours worked per week" are at least 35 hours and exclude individuals living in group quarters (Ruggles et al. 2023). I classify workers who have completed at least four years of college as college workers and all other workers as non-college workers.

Table 3 displays the distribution of population, the college-employment ratio, and per capita income in 2016. The largest MSA in 2016 in this sample is New York-Newark-Jersey City (NY-NJ-PA), which had a population of 19,943,198, while the smallest is Parkersburg-Vienna, WV, which had a population of 91,940. The college employment ratio and per capita personal income have substantial variations across MSAs. The table also shows their correlations to population. The high value of the college employment ratio validates the concern of the alternative explanation by skill-labor supply in the robustness checks.

Table 3: Distribution of Population, College-Employment Ratio, and Per Capita Personal Income across MSAs in 2016

| | Min | Q1 | Median | Q3 | Max | Corr to pop. |
|---|---|---|---|---|---|---|
| Population | 91,940 | 174,538 | 380,010 | 847,835 | 19,943,198 | 1.00 |
| College-employment ratio | 0.19 | 0.40 | 0.54 | 0.69 | 1.68 | 0.41 |
| Per capita personal income | 25,593 | 38,812 | 43,289 | 49,803 | 10,6272 | 0.43 |

*Notes:* Correlation is calculated after taking logs of both variables.

Table 4 summarizes the elasticities of employment with respect to the MSA's population I estimated and the income elasticities I borrow from Caron et al. (2020).

Table 4: Estimates for Sectors

| GTAP code | Sector | Elasticity of employment to MSA's population | Income elasticity of demand |
|---|---|---|---|
| atp | Air transport | 1.52 | 1.06 |
| ppp | Paper prod., publishing | 1.19 | 1.07 |
| obs | Business services | 1.17 | 1.21 |
| cmn | Communication | 1.16 | 1.17 |
| ofi | Fin. Services nec | 1.16 | 1.13 |
| tex | Textiles | 1.14 | 0.77 |
| ros | Recreational and other services | 1.10 | 1.17 |
| wtp | Water transport | 1.10 | 1.05 |
| isr | Insurance | 1.05 | 1.41 |
| cns | Construction | 1.03 | 0.83 |
| ele | Electronic equipment | 1.02 | 1.21 |
| otp | Transport nec | 1.02 | 0.83 |
| trd | Trade | 1.01 | 1.09 |
| osg | Pub. Adm. and services | 1.00 | 0.99 |
| ely | Electricity | 0.96 | 0.94 |
| omf | Manuf. nec | 0.95 | 1.02 |
| p_c | Petroleum, coal prod. | 0.93 | 0.78 |
| ome | Machinery and equipment nec | 0.92 | 0.93 |
| omt+cmt | Meat prod. | 0.92 | 1.06 |
| crp | Chemical, rubber, plastic products | 0.91 | 0.91 |
| wtr | Water | 0.90 | 0.97 |
| ofd | Food prod. nec | 0.89 | 0.83 |
| lea | Leather prod. | 0.86 | 1.02 |
| otn | Transport equipment nec | 0.85 | 1.03 |
| b_t | Beverage and tobacco prod. | 0.70 | 0.76 |
| mvh | Motor vehicles and parts | 0.69 | 1.06 |
| lum | Wood prod. | 0.65 | 1.07 |
| mil | Dairy prod. | 0.58 | 1.02 |
| frs | Forestry | 0.19 | 0.36 |

*Notes:* Adm., bev., fin., equip., mach., manuf., prod., and pub., stand for administration, beverage, financial, equipment, machinery, manufactures, products, and public, respectively.

# B Derivation of Theoretical Results

## B.1 Workers' Problem

Given a city to reside in, the consumption optimization problem is given by

$$\begin{aligned}\max_{C_n,\{Q_{nk}\}_{k\in\mathcal{K}},\{q_{nk}(\nu)\}_{\nu\in\Omega_{nk},k\in\mathcal{K}}} \quad & C_n + \lambda\left(C_n - \left[\sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}\right) \\ & + \sum_{k\in\mathcal{K}} \xi_k \left(Q_{nk} - \left[\int_{\Omega_{nk}} q_{nk}(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}}\right) \\ & + \omega\left(E_n - \sum_{k\in\mathcal{K}} \int_{\Omega_{nk}} p_{nk}(\nu) q_{nk}(\nu) d\nu\right),\end{aligned}$$

where $\lambda$, $\{\xi_k\}_{k\in\mathcal{K}}$, and $\omega$ are the Lagrange multipliers. The first order conditions (FOCs) are as follows:

$$\begin{aligned} C_n : 1 + \lambda &= \lambda \frac{\eta}{\eta-1}\left(\sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1}\left(\sum_{k\in\mathcal{K}} \frac{\epsilon_k}{\eta} \beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} C_n^{-\frac{1}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}}\right), \\ Q_{nk} : \lambda &\left(\sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}-1}\left(\beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}-1}\right) = \xi_k, \\ q_{nk}(\nu) : &\xi_k \left[\int_{\Omega_{nk}} q_{nk}(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}-1} q_{nk}(\nu)^{\frac{\sigma-1}{\sigma}-1} = \omega p_{nk}(\nu), \\ \lambda : &C_n = \left[\sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \\ \xi_k : &Q_{nk} = \left[\int_{\Omega_{nk}} q_{nk}(\nu)^{\frac{\sigma-1}{\sigma}} d\nu\right]^{\frac{\sigma}{\sigma-1}} \\ \omega : &E_n = \sum_{k\in\mathcal{K}} \int_{\Omega_{nk}} p_{nk}(\nu) q_{nk}(\nu) d\nu \end{aligned}$$

First, I derive the usual result with constant elasticity of substitution from the FOC with respect to (w.r.t.) $q_{nk}(\nu)$ and that w.r.t. $\xi_k$.

$$P_{nk} Q_{nk}^{\frac{1}{\sigma}} = p_{nk}(\nu) q_{nk}(\nu)^{\frac{1}{\sigma}},$$

where $P_{nk} = \left[\int_{\Omega_{nk}} (p_{nk}(\nu))^{1-\sigma} d\nu\right]^{1/(1-\sigma)}$. It follows from this result, the FOC w.r.t $Q_{nk}$, and that w.r.t $q_{nk}(\nu)$,

$$\lambda \frac{\beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}-1}}{\left(\sum_{\ell\in\mathcal{K}} \beta_\ell^{\frac{1}{\eta}} C_n^{\frac{\epsilon(\ell)}{\eta}} Q_{n\ell}^{\frac{\eta-1}{\eta}}\right)^{\frac{1}{\eta-1}}} = \omega P_{nk} \implies \frac{\beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}}}{\beta_\ell^{\frac{1}{\eta}} C_n^{\frac{\epsilon(\ell)}{\eta}} Q_{n\ell}^{\frac{\eta-1}{\eta}}} = \frac{P_{nk}Q_{nk}}{P_{n\ell}Q_{n\ell}}. \tag{17}$$

Next, I obtain the expenditure by using eq. (17).

$$\begin{aligned} E_n = \sum_{k\in\mathcal{K}} \int_{\Omega_{nk}} p_{nk}(\nu) q_{nk}(\nu) d\nu &= \sum_{k\in\mathcal{K}} P_{nk}Q_{nk} \\ &= \sum_{k\in\mathcal{K}} P_{n\ell}Q_{n\ell} \frac{\beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}}}{\beta_\ell^{\frac{1}{\eta}} C_n^{\frac{\epsilon(\ell)}{\eta}} Q_{n\ell}^{\frac{\eta-1}{\eta}}} \\ &= \frac{P_{n\ell}Q_{n\ell}}{\beta_\ell^{\frac{1}{\eta}} C_n^{\frac{\epsilon(\ell)}{\eta}} Q_{n\ell}^{\frac{\eta-1}{\eta}}} \sum_{k\in\mathcal{K}} \beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} Q_{nk}^{\frac{\eta-1}{\eta}} \\ &= \frac{P_{n\ell}Q_{n\ell}}{\beta_\ell^{\frac{1}{\eta}} C_n^{\frac{\epsilon(\ell)}{\eta}} Q_{n\ell}^{\frac{\eta-1}{\eta}}} C_n^{\frac{\eta-1}{\eta}}, \end{aligned}$$

where the third equality follows from (17) and the last from the FOC w.r.t. $\lambda$. The demand function immediately follows.

$$Q_{nk} = \beta_k C_n^{\epsilon(k)} P_{nk}^{-\eta} C_n^{1-\eta} E_n^{\eta}. \tag{18}$$

Subsequently, the expenditure share (eq. (3)) follows.

$$s_{nk} = \frac{\beta_k C_n^{\epsilon(k)} P_{nk}^{1-\eta}}{\sum_{k\in\mathcal{K}} \beta_\ell C_n^{\epsilon(\ell)} P_{n\ell}^{1-\eta}}.$$

Substituting eq. (18) for $Q_{nk}$ in the FOC w.r.t. $\lambda$ yields indirect real income.

$$C_n = \left[ \sum_{k \in \mathcal{K}} \beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} \left( \beta_k^{\frac{1}{\eta}} C_n^{\frac{\epsilon(k)}{\eta}} P_{nk}^{-1} U_n^{\frac{1-\eta}{\eta}} E_n \right)^{\eta - 1} \right]^{\frac{\eta}{\eta-1}}$$

$$\iff C_n = \left( \sum_{k \in \mathcal{K}} \beta_k C_n^{\epsilon(k)} P_{nk}^{1-\eta} \right)^{-\frac{1}{1-\eta}} E_n$$

## B.2 Firms' Problem and Labor Demand

I begin by substituting the optimized production into the zero-profit condition. As for the optimized production, the optimized price for a firm in city $n$ to sell in city $i$ is as follows.

$$\forall k, \forall n, \ p_{ni,k} = \begin{cases} \frac{\sigma}{\sigma-1} \frac{v_k}{\lambda_n} w_n & i = n \\ \tau_k \frac{\sigma}{\sigma-1} \frac{v_k}{\lambda_n} w_n & i \neq n. \end{cases}$$

Subsequently, the profit becomes

$$\pi_{nk} = \frac{1}{\sigma - 1} \frac{v_k}{\lambda_n} q_{nn,k} + \tau_k \sum_{i \neq n} \frac{1}{\sigma - 1} \frac{v_k}{\lambda_n} q_{ni,k} - \frac{f_k}{\lambda_n}.$$

The zero-profit condition requires this $\pi_{nk}$ to be zero, yielding the labor demand per firm as follows:

$$\left( q_{nn,k} + \tau_k \sum_{i \neq n} q_{ni,k} \right) \frac{v_k}{\lambda_n} + \frac{f_k}{\lambda_n} = \sigma \frac{f_k}{\lambda_n}. \quad (19)$$

The fixed cost and productivity levels determine the labor demand. Now, I use normalization. It can be shown that $\beta_k$, $f_k$, and $v_k$ affect the equilibrium values of endogenous variables only through $\beta_k^{\frac{1}{1-\eta}} f_k^{\frac{1}{\sigma-1}} v_k$. I set $v_k = (\sigma - 1)/\sigma$ and $f_k = 1/\sigma$ so that $p_{nk} = w_n/\lambda_n$ and the labor demand per firm is $1/\lambda_n$. I replace $\beta_k$ by $\tilde{\beta}_k = \beta_k \left( f_k^{\frac{1}{\sigma-1}} v_k \sigma^{\sigma/(\sigma-1)} (\sigma - 1)^{-1} \right)^{1-\eta}$ so that this change does not affect the equilibrium values of endogenous variables. Thus, it follows from eq. (19) and the normalization that the aggregate supply of goods by a firm is given by

$$\forall k,\, q_{nn,k} + \tau_k \sum_{i \neq n} q_{ni,k} = 1.$$

This supply is equal to the demand in equilibrium as follows:

$$\begin{aligned} 1 &= L_n p_{nn,k}^{-\sigma} P_{nk}^{\sigma-1} E_n s_{nk} + \tau_k \sum_{i \neq n} L_i \left(\tau_k p_{ni,k}\right)^{-\sigma} P_{ik}^{\sigma-1} E_i s_{ik} \\ &= L_n \left(\frac{w_n}{\lambda_n}\right)^{-\sigma} P_{nk}^{\sigma-1} E_n s_{nk} + \tau_k \sum_{i \neq n} L_i \left(\tau_k \frac{w_n}{\lambda_n}\right)^{-\sigma} P_{ik}^{\sigma-1} E_i s_{ik}. \end{aligned} \tag{20}$$

This equation reflects the zero-profit condition and holds for all $n$ for each sector $k$. I use a matrix $R_k$ that is defined by

$$R_k = (1 - \phi_k) I + \phi_k S,$$

where $I$ is the identity matrix of size $N$, and $S$ is a square matrix of size $N$ in which all the entries equal to 1. The inverse matrix is

$$R_k^{-1} = \frac{1}{1 - \phi_k} I - \frac{1}{(1 - \phi_k)(1/\phi_k + N - 1)} S.$$

Given $R_k$, eq. (20) for all $n$ can be summarized as

$$R_k \begin{bmatrix} L_1 P_{1k}^{\sigma-1} E_1 s_{1k} \\ L_2 P_{2k}^{\sigma-1} E_2 s_{2k} \\ \vdots \\ L_N P_{Nk}^{\sigma-1} E_n s_{Nk} \end{bmatrix} = \begin{bmatrix} \lambda_1^{-\sigma} w_1^{\sigma} \\ \lambda_2^{-\sigma} w_2^{\sigma} \\ \vdots \\ \lambda_N^{-\sigma} w_N^{\sigma} \end{bmatrix}.$$

Premultiplying $R_k^{-1}$ yields

$$L_n E_n s_{nk} \cdot P_{nk}^{\sigma-1} = \frac{(w_n/\lambda_n)^{\sigma}}{1 - \phi_k} - \frac{1}{(1 - \phi_k)(1/\phi_k + N - 1)} \sum_{i \in N} (w_i/\lambda_i)^{\sigma},$$

from which eq. (7) in the main text follows. Substituting $(\lambda_{nk} x_{nk} L_n) \cdot (w_n/\lambda_n)^{1-\sigma} + \sum_{n \neq i} (\lambda_{ik} x_{ik} L_i) \cdot (\tau_k w_i/\lambda_{ik})^{1-\sigma}$ for $P_{nk}^{1-\sigma}$ makes the matrix form be

$$\begin{bmatrix} w_1 s_{1k} L_1 \\ \vdots \\ w_N s_{Nk} L_N \end{bmatrix} = diag\left( R_k \begin{bmatrix} x_{1k} \lambda_1^{\sigma} L_1 w_1^{1-\sigma} \\ \vdots \\ x_{Nk} \lambda_N^{\sigma} L_N w_N^{1-\sigma} \end{bmatrix} \right) \cdot R_k^{-1} \begin{bmatrix} \lambda_1^{-\sigma} w_1^{\sigma} \\ \lambda_2^{-\sigma} w_2^{\sigma} \\ \vdots \\ \lambda_N^{-\sigma} w_N^{\sigma} \end{bmatrix},$$

where I used $E_n = w_n$. Further manipulations yield

$$\begin{bmatrix} w_1 s_{1k} L_1 \\ \vdots \\ w_N s_{Nk} L_N \end{bmatrix} = diag\left( R_k^{-1} \begin{bmatrix} \lambda_1^{-\sigma} w_1^{\sigma} \\ \lambda_2^{-\sigma} w_2^{\sigma} \\ \vdots \\ \lambda_N^{-\sigma} w_N^{\sigma} \end{bmatrix} \right) R_k \begin{bmatrix} x_{1k} \lambda_1^{\sigma} L_1 w_1^{1-\sigma} \\ \vdots \\ x_{Nk} \lambda_N^{\sigma} L_N w_N^{1-\sigma} \end{bmatrix}$$

$$\Longrightarrow \begin{bmatrix} x_{1k} L_1 w_1^{1-\sigma} \\ \vdots \\ x_{Nk} L_N w_N^{1-\sigma} \end{bmatrix} = \begin{bmatrix} \lambda_1^{-\sigma} \frac{s_{1k} L_1 w_1}{\lambda_{1k}^{-\sigma} w_1^{\sigma} - \frac{\sum_n \lambda_n^{-\sigma} w_n^{\sigma}}{(1/\phi_k + N - 1)}} - \lambda_1^{-\sigma} \frac{1}{1/\phi_k + N - 1} \sum_i \frac{s_{ik} L_i w_i}{\lambda_i^{-\sigma} w_i^{\sigma} - \frac{\sum_n \lambda_n^{-\sigma} w_n^{\sigma}}{1/\phi_k + N - 1}} \\ \vdots \\ \lambda_N^{-\sigma} \frac{s_{Nk} L_N w_N}{\lambda_N^{-\sigma} w_N^{\sigma} - \frac{\sum_n \lambda_n^{-\sigma} w_n^{\sigma}}{(1/\phi_k + N - 1)}} - \lambda_N^{-\sigma} \frac{1}{1/\phi_k + N - 1} \sum_i \frac{s_{ik} L_i w_i}{\lambda_i^{-\sigma} w_i^{\sigma} - \frac{\sum_n \lambda_n^{-\sigma} w_n^{\sigma}}{(1/\phi_k + N - 1)}} \end{bmatrix}.$$

Eq. (10) in the main text immediately follows from this matrix form. Premultiplying the matrix above by the row matrix $[1, 1, \dots, 1]$ with $\sum_{k \in \mathcal{K}} x_{nk} = \sum_{k \in \mathcal{K}} s_{nk} = 1$ yields

$$L_n w_n = \left(1 + \sum_{k \in \mathcal{K}} \mu_{nk} s_{nk}\right) L_n w_n - \sum_{k \in \mathcal{K}} \frac{(w_n/\lambda_n)^{\sigma}}{(\phi_k^{-1} + N - 1)} \sum_{i \in \mathcal{N}} \frac{L_i w_i}{(w_i/\lambda_i)^{\sigma}} (1 + \mu_{ik}) s_{ik}$$

$$\iff (\sum_{k \in \mathcal{K}} \mu_{nk} s_{nk}) L_n w_n = (w_n/\lambda_n)^{\sigma} \sum_{k \in \mathcal{K}} \frac{1}{(\phi_k^{-1} + N - 1)} \sum_{i \in \mathcal{N}} \frac{L_i w_i}{(w_i/\lambda_i)^{\sigma}} (1 + \mu_{ik}) s_{ik}$$

$$\iff \frac{(w_n/\lambda_n)^{\sigma}}{\sum_{j \in \mathcal{N}} (w_j/\lambda_j)^{\sigma}} = \frac{\left(\sum_{k \in K} \mu_{nk} s_{nk}\right) L_n \lambda_n (w_n/\lambda_n)}{\sum_{i \in \mathcal{N}} \left(\sum_{k \in K} \mu_{ik} s_{ik}\right) L_i \lambda_i (w_i/\lambda_i)}.$$

This is the city-level labor demand (11).

## B.3 Derivation of Expenditure Shares and Labor Supply

I start with dividing both sides of the zero-profit condition (7) by $w^\sigma$,

$$L_n s_{nk} \cdot \left(\frac{P_{nk}}{w_n}\right)^{\sigma-1} = \frac{\lambda_n^{-\sigma}}{1-\phi_k} - \frac{1}{w_n^\sigma} \frac{\sum_{i\in N}(w_i/\lambda_i)^\sigma}{(1-\phi_k)(1/\phi_k+N-1)}, \tag{21}$$

where I used $E_n = w_n$. I solve for an expenditure share after substituting eq. (3) for $P_{nk}$ in eq. (21) with $P_n = E_n/C_n$.

$$s_{nk} = \left\{ \frac{1}{(1-\phi_k)(1+\mu_{nk})} \left( \tilde{\beta}_k^{\frac{1}{1-\eta}} C_n^{\frac{\epsilon(k)}{1-\eta}} \frac{C_n}{\lambda_n (\lambda_n L_n)^{\frac{1}{\sigma-1}}} \right)^{\sigma-1} \right\}^{\frac{1-\eta}{\sigma-\eta}},$$

which is eq. (12).

I obtain an equation expressing city-level labor supply without employment shares to prove propositions. Aggregating eq. (12) over sectors yields

$$C_n = \lambda_n \left(\lambda_n L_n\right)^{\frac{1}{\sigma-1}} \left[ \sum_{k\in\mathcal{K}} \left\{ \frac{\tilde{\beta}_k^{\frac{\sigma-1}{1-\eta}} C_n^{\epsilon(k)\frac{\sigma-1}{1-\eta}}}{(1-\phi_k)(1+\mu_{nk})} \right\}^{\frac{1-\eta}{\sigma-\eta}} \right]^{-\frac{\sigma-\eta}{1-\eta}\cdot\frac{1}{\sigma-1}}. \tag{22}$$

This equation, unlike eq. (14) in the main text, expresses real income in city $n$ as a function of the city size and wages because the zero-profit conditions removed employment shares. The first and second factors in the right-hand side reflect productivity in producing a variety and the mass of available varieties from efficiency units of labor, respectively. [31] Combining eq. (22) with population allocation with Fréchet utility shocks (eq. (4)) yields

$$1 = \sum_{k\in\mathcal{K}} \left\{ \frac{\tilde{\beta}_k^{\frac{\sigma-1}{1-\eta}} C_1^{(\frac{\epsilon(k)}{1-\eta}+1)(\sigma-1)}}{(1-\phi_k)(1+\mu_{nk})L_1} \left[ \left(\frac{L_n}{L_1}\right)^\gamma a_n^{-1} \right]^{(\frac{\epsilon(k)}{1-\eta}+1)(\sigma-1)} \left(\frac{L_n}{L_1}\right)^{-1} \lambda_n^{-\sigma} \right\}^{\frac{1-\eta}{\sigma-\eta}}, \tag{23}$$

which uses city 1 as the base. Recall $\partial\mu_{nk}/\partial(w_n/\lambda_n) < 0$. Given Assumption 1, a city size $L_n$ and factor

[31] The right-hand side has $C_n^{\epsilon(k)\frac{\sigma-1}{1-\eta}}$, but $\min_{k\in\mathcal{K}}\{\epsilon(k)/(1-\eta)\} > -1$ ensures that $C_n$ in the left-hand side dominates $C_n^{\epsilon(k)\frac{\sigma-1}{1-\eta}}$, allowing me to ignore $C_n^{\epsilon(k)\frac{\sigma-1}{1-\eta}}$ when evaluating the directions of changes.

price $w_n/\lambda_n$ are in an inverse relationship in eq. (23).[32]

## B.4 Proofs of Theorem, Propositions and Lemmas

*Proof of Proposition 1.* It follows from equations (9) and (11) for two cities. □

*Proof of Theorem 1.* Given wages, $\sum_{k\in K}\mu_{nk}s^a_{nk}\geq\sum_{k\in K}\mu_{nk}s^b_{nk}$. Subsequently, equations (9) and (11) for the two cities imply Theorem 1. □

*Proof of Lemma 1.* Given common tradabiliy, the city-level labor demand (11) becomes

$$\frac{(w_n/\lambda_n)^\sigma}{\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}=\frac{\mu_n L_n\lambda_n(w_n/\lambda_n)}{\sum_{i\in\mathcal{N}}\mu_i L_i\lambda_i(w_i/\lambda_i)}, \tag{24}$$

where $\mu_i=\left[\frac{(\phi^{-1}+N-1)(w_i/\lambda_i)^\sigma}{\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}-1\right]^{-1}$ and $\sum_{k\in\mathcal{K}}m_k=1$. This equation holds for each city, but one of them is redundant because of Walras's law. Dividing each one by that of city 1 yields

$$1=\left[\frac{(\phi^{-1}+N-1)-\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}{(\phi^{-1}+N-1)(w_i/\lambda_i)^\sigma-\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}\right]\lambda_n\left(\frac{w_n}{\lambda_n}\right)^{1-\sigma}\frac{L_n}{L_1}. \tag{25}$$

I show that there exists $\{C_1, L_1, \{w_n, L_n\}_{n=2,\dots N}\}$ that solves the system of eq. (23) for $n=1,\dots,N$ and (25) for $n=2,\dots,N$, and $\sum L_n=L$ by Brouwer's fixed point theorem. To do so, I use Lemma 8.

**Lemma 8.** *Suppose* $\min_k(a_n^{-1}\left(\lambda_n^{-1}\right)^{\gamma+\left(\frac{\epsilon(k)}{1-\eta}+1\right)^{-1}}>1$*. Then, given* $(L_1,\Psi)\in[L/\sum_{n\in\mathcal{N}}\lambda_n^{-1},L]\times[1+\phi(N-1),N]$*, there exits unique* $(C_1, L_n/L_1, w_n)\in\mathbb{R}_{++}\times(0,\lambda_n^{-1})\times(\underline{w_n},\lambda_n)$ *where* $\underline{w_n}=[\Psi(\phi^{-1}+$

[32] Although this relationship sounds counterintuitive, eq. (22) considers endogenous changes in price indices that a wage change entails; namely, a higher input cost necessitates more-than-proportional increases of the price indices to offset the disadvantage for local firms, hurting households.

$N-1)^{-1}]^{1/\sigma}\lambda_n$ *that solves the system of equations:*

$$1 = \sum_{k\in\mathcal{K}} W_k(C_1, L_1, \Psi), \tag{26}$$

$$1 = \sum_{k\in\mathcal{K}} W_k(C_1, L_1, \Psi) \cdot \left\{ \left(\frac{L_n}{L_1}\right)^{\gamma(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)-1} a_n^{-(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)} \lambda_n^{-\sigma} \left[\frac{\phi^{-1}+N-1-\Psi(w_n/\lambda_n)^{-\sigma}}{\phi^{-1}+N-1-\Psi}\right]\right\}^{\frac{1-\eta}{\sigma-\eta}}, \tag{27}$$

$$1 = \left[\frac{\phi^{-1}+N-1-\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^{\sigma}}{\phi^{-1}+N-1-(w_n/\lambda_n)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^{\sigma}}\right] \lambda_n \left(\frac{w_n}{\lambda_n}\right)^{1-2\sigma} \frac{L_n}{L_1}, \tag{28}$$

*where* $W_k(C_1, L_1, \Psi) = \left\{ \frac{\tilde{\beta}_k^{\frac{\sigma-1}{1-\eta}} C_1^{(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)}}{(1-\phi)L_1} \left[1 - \frac{\Psi}{\phi^{-1}+N-1}\right]\right\}^{\frac{1-\eta}{\sigma-\eta}}.$

*Proof.* Eq.(26) has a unique solution $C_1$. Given this $C_1$ and $w_n \in (\underline{w_n}, \lambda_n]$, I let $L_n/L_1 = f_1^n(w_n)$ be the solution to eq. (27) and $L_n/L_1 = f_2^n(w_n)$ be that to eq. (28). I show that there exists unique $w_n \in (\underline{w_n}, \lambda_n)$ that solves $f_1^n(w_n) = f_2^n(w_n)$. Given Assumption 1, $\lim_{w_n\to\underline{w_n}} f_1^n(w_n) = \infty > \lim_{w_n\to\underline{w_n}} f_2^n(w_n) = 0$. At $w_n = \lambda_n$, $f_2^n(\lambda_n) = \lambda_n^{-1}$, and eqs. (26) and (27) imply

$$\begin{aligned}
&\min_k \left(f_1^n(\lambda_n)\right)^{\gamma(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)-1} a_n^{-(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)} \lambda_n^{-\sigma} \leq 1 \\
\Longrightarrow\ &\min_k \left(\lambda_n f_1^n(\lambda_n)\right)^{\gamma(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)-1} < 1 \ \left(\because \min_k a_n^{-1}\left(\lambda_n^{-1}\right)^{\gamma+\left(\frac{\epsilon(k)}{1-\eta}+1\right)^{-1}} > 1\right) \\
\Longleftrightarrow\ & f_1^n(\lambda_n) < \lambda_n^{-1} \iff f_1^n(\lambda_n) < f_2^n(\lambda_n).
\end{aligned}$$

$f_1^n(w_n)$ and $f_2^n(w)$ are continuous on $(\underline{w_n}, \lambda_n)$. Thus, there exists $w_n \in (\underline{w_n}, \lambda_n)$ such that $f_1^n(w_n) = f_2^n(w_n)$ by the intermediate value theorem, and it is unique because $f_1^n(w_n)$ and $f_2^n(w_n)$ decreases and increases, respectively, in $w_n$. Further, $0 < f_1(\lambda_n^n) < L_n/L_1 < f_2^n(\lambda_n) = \lambda_n^{-1}$. □

Given Lemma 8, I let $f_3$ map $(L_1, \Psi) \in [L/\sum_{n\in\mathcal{N}}\lambda_n^{-1}, L] \times [1+\phi(N-1), N]$ to $(L_1', \Psi')$ such that $L_1' = L/\sum_{n\in\mathcal{N}}(L_n/L_1)$ and $\Psi' = \sum_{n\in\mathcal{N}}(w_n/\lambda_n)^{\sigma}$, where $L_n/L_1$ and $w_n$ form the unique solution in Lemma 8. Lemma 8 implies $L/(\sum_{n\in\mathcal{N}}\lambda_n^{-1}) < L_1' < L$ and $1+\phi(N-1) < \Psi' < N$. Thus, $f_3$ is a map from $[L/\sum_{n\in\mathcal{N}}\lambda_n^{-1}, L] \times [1+\phi(N-1), N]$ to itself, and it has a fixed point by Brouwer's fixed point theorem. Given the fixed point $(L_1, \sum_{n\in\mathcal{N}}(w_n/\lambda_n)^\sigma)$, the solution to the system of equations in Lemma 8 provide $C_1$, $\{L_n/L_1,\ w_n\}_{n=2\ldots N}$ satisfying eq. (23) for $n=1,\ldots,N$ and (25) for $n=2,\ldots,N$, and $\sum_{n\in\mathcal{N}} L_n = L$. $\square$

*Proof of Proposition 2.* I prove $L_n > L_i$ by contradiction. Suppose $L_n < L_i$. Then, eqs. (29), (30), and (31) imply

$$1 > \min_k \left(\frac{a_i}{a_n}\right)^{-(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)} \left(\frac{\lambda_i}{\lambda_n}\right)^{-\sigma} \left[\frac{\phi^{-1}+N-1-(w_i/\lambda_i)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}{\phi^{-1}+N-1-(w_n/\lambda_n)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}\right],$$
$$1 > \left[\frac{\phi^{-1}+N-1-(w_n/\lambda_n)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}{\phi^{-1}+N-1-(w_i/\lambda_i)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}\right] \frac{\lambda_i}{\lambda_n}\left(\frac{w_i}{\lambda_i}/\frac{w_n}{\lambda_n}\right)^{1-2\sigma}.$$

Given $\min_k (a_n/a_i)\,(\lambda_n/\lambda_i)^{(1+(\sigma-1)^{-1})\theta(k)} > 1$, the first inequality implies $w_i/\lambda_i < w_n/\lambda_n$. Canceling out the term in square brackets yields

$$1 > \min_k \left(\frac{a_i}{a_n}\right)^{-(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)} \left(\frac{\lambda_i}{\lambda_n}\right)^{1-\sigma} \left(\frac{w_i}{\lambda_i}/\frac{w_n}{\lambda_n}\right)^{1-2\sigma}$$
$$> \min_k \left(\frac{a_i}{a_n}\right)^{-(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)} \left(\frac{\lambda_i}{\lambda_n}\right)^{1-\sigma},$$

where the second inequality follows from $w_i/\lambda_i < w_n/\lambda_n$. This inequality contradicts $\min_{k\in\mathcal{K}}(a_n/a_i)\,(\lambda_n/\lambda_i)^{\theta(k)} > 1$. $\square$

*Proof of Proposition 3.* Eqs. (23) and (25) for cities $n$ and $i$ can be rewritten as

$$1 = \sum_{k\in\mathcal{K}} W_{nk}, \tag{29}$$

$$1 = \sum_{k\in\mathcal{K}} W_{nk} \left\{ \left(\frac{L_i}{L_n}\right)^{\gamma(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)-1} \left(\frac{a_i}{a_n}\right)^{-(\sigma-1)\left(\frac{\epsilon_k}{1-\eta}+1\right)} \left(\frac{\lambda_i}{\lambda_n}\right)^{-\sigma} \right\}^{\frac{1-\eta}{\sigma-\eta}} \cdot \left\{\left[\frac{\phi^{-1}+N-1-(w_i/\lambda_i)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}{\phi^{-1}+N-1-(w_n/\lambda_n)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}\right]\right\}^{\frac{1-\eta}{\sigma-\eta}}, \tag{30}$$

$$1 = \left[\frac{\phi^{-1}+N-1-(w_n/\lambda_n)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}{\phi^{-1}+N-1-(w_i/\lambda_i)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}\right] \frac{\lambda_i}{\lambda_n}\left(\frac{w_i}{\lambda_i}/\frac{w_n}{\lambda_n}\right)^{1-2\sigma} \frac{L_i}{L_n}, \tag{31}$$

where $W_{nk} = \left\{ \frac{\tilde{\beta}_k^{\frac{\sigma-1}{1-\eta}} C_n^{(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)}}{(1-\phi)L_n} \left[ 1 - \frac{(w_n/\lambda_n)^{-\sigma} \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma}{\phi^{-1}+N-1} \right] \right\}^{\frac{1-\eta}{\sigma-\eta}}$. Given that $\min_{k\in\mathcal{K}}(a_n/a_i)(\lambda_n/\lambda_i)^{\gamma+\theta(k)} > 1$, the same steps as Lemma 8 lead to $[(\phi^{-1}+N-1)^{-1}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma]^{1/\sigma}\lambda_i < w_i < \lambda_i w_n/\lambda_n$. □

*Proof of Proposition 4.* Substituting eq. (4) for real income in the sectoral expenditure share ratio that follows from eq. (12) yields $(s_{nk}/s_{ik})\,/\,(s_{n\ell}/s_{i\ell}) = (L_n/L_i)^{\gamma(\frac{\sigma-1}{\sigma-\eta})(\epsilon(k)-\epsilon(\ell))}$. □

*Proof of Corollary 2.* It follows from Proposition 4 and Lemma 2. □

*Proof of Lemma 3.* The equilibrium conditions are eq. (12) for each city-sector pair, eqs. (11) and (23) for each city, and $\sum_{\mathcal{N}} L_n = L$. Eq. (11) for one city is redundant because of Walras's law. I divide eq. (11) by that of city 1 and substitute eq. (12) for $\{s_{nk}\}_{n\in\mathcal{N},k\in\mathcal{K}}$; then, for city $n$

$$\left(\frac{w_n}{\lambda_n}\right)^{2\sigma-1} \lambda_n^{-\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta} = \sum_{k\in K} X_k\left(C_1, L_1, \sum_{j\in\mathcal{N}}\left(\frac{w_j}{\lambda_j}\right)^\sigma\right) \frac{(1+\mu_{nk})^{\frac{\sigma-1}{\sigma-\eta}}}{(1+\mu_{1k})^{\frac{\sigma-1}{\sigma-\eta}}} \left(\frac{C_n}{C_1}\right)^{\frac{1-\eta}{\sigma-\eta}(\sigma-1+\frac{\epsilon(k)}{1-\eta})} \left(\frac{L_n}{L_1}\right)^{\frac{\sigma-1}{\sigma-\eta}},$$

where

$$X_k\left(C_1, L_1, \sum_{j\in\mathcal{N}}\left(\frac{w_j}{\lambda_j}\right)^\sigma\right) = \frac{\mu_{1k}s_{1k}}{\sum_{k\in K}\mu_{1k}s_{1k}}$$

$$= \frac{\left[\frac{\phi_k^{-1}+N-1}{\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma} - 1\right]^{-1} \left\{ (1-\phi_k)^{-1} \left( \tilde{\beta}_k^{\frac{1}{1-\eta}} \frac{C_1^{\frac{\epsilon(k)}{1-\eta}+1}}{L_1^{\frac{1}{\sigma-1}}} \right)^{\sigma-1} \left(1 - \frac{\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}{\phi_k^{-1}+N-1}\right) \right\}^{\frac{1-\eta}{\sigma-\eta}}}{\sum_{k\in\mathcal{K}} \left[\frac{\phi_k^{-1}+N-1}{\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma} - 1\right]^{-1} \left\{ (1-\phi_k)^{-1} \left( \tilde{\beta}_k^{\frac{1}{1-\eta}} \frac{C_1^{\frac{\epsilon(k)}{1-\eta}+1}}{L_1^{\frac{1}{\sigma-1}}} \right)^{\sigma-1} \left(1 - \frac{\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}{\phi_k^{-1}+N-1}\right) \right\}^{\frac{1-\eta}{\sigma-\eta}}}.$$

Substituting eq. (4) for $C_n/C_1$ yields

$$\left(\frac{w_n}{\lambda_n}\right)^{2\sigma-1} \lambda_n^{-\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta} = \sum_{k\in\mathcal{K}} X_k\left(C_1, L_1, \sum_{j\in\mathcal{N}}\left(\frac{w_j}{\lambda_j}\right)^\sigma\right) \cdot \left(\frac{\phi_k^{-1}+N-1-\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}{\phi_k^{-1}+N-1-(w_n/\lambda_n)^{-\sigma}\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^\sigma}\right)^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{L_n}{L_1}\right)^{\left[\gamma(\frac{\epsilon(k)}{1-\eta}+1)(1-\eta)+1\right]\frac{\sigma-1}{\sigma-\eta}}. \quad (32)$$

Given Assumption 4, $w_n$ increases with $L_n/L_1$, as analyzed in Section 3.

I show that there exists $\{C_1, L_1, \{w_n, L_n\}_{n=2,\dots N}\}$ that solves the system of eq. (23) for all $n$ and (32) for $n = 2, \dots, N$, and $\sum_n L_n = L$ by Brouwer's fixed point theorem. I start with Lemma 9.

**Lemma 9.** *Given* $(L_1, \Psi) \in [L/\sum_{n\in\mathcal{N}} \bar{\ell}_n, L] \times [1 + \max_k \phi_k(N-1), N]$ *where* $\bar{\ell}_n = \lambda_n^{\chi}$*, and* $\chi = \frac{1-\frac{1}{1-\eta}}{\gamma(\frac{\min_k \epsilon(k)}{1-\eta}+1)+\frac{1}{1-\eta}}$*, there exits unique* $(C_1, L_n/L_1, w_n) \in \mathbb{R}_{++}\times(0, \bar{\ell}_n]\times(\underline{w_n}, \lambda_n)$ *where* $\underline{w_n} = [\Psi((\max_k \phi_k)^{-1} + N-1)^{-1}]^{1/\sigma}\lambda_n$ *that solves the system of equations:*

$$1 = \sum_{k\in\mathcal{K}} W_k(C_1, L_1, \Psi), \tag{33}$$

$$1 = \sum_{k\in\mathcal{K}} W_k(C_1, L_1, \Psi) \left\{ \left(\frac{L_n}{L_1}\right)^{\gamma(\sigma-1)\left(\frac{\epsilon_k}{1-\eta}+1\right)-1} \lambda_n^{-\sigma} \left[\frac{\phi_k^{-1} + N - 1 - \Psi(w_n/\lambda_n)^{-\sigma}}{\phi_k^{-1} + N - 1 - \Psi}\right]\right\}^{\frac{1-\eta}{\sigma-\eta}}, \tag{34}$$

$$\begin{aligned} 1 = &\left(\frac{w_n}{\lambda_n}\right)^{1-2\sigma} \lambda_n^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta} \\ &\cdot \sum_{k\in\mathcal{K}} X_k(C_1, L_1, \Psi) \left(\frac{\phi_k^{-1} + N - 1 - \Psi}{\phi_k^{-1} + N - 1 - \Psi(w_n/\lambda_n)^{-\sigma}}\right)^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{L_n}{L_1}\right)^{\left[\gamma(\frac{\epsilon(k)}{1-\eta}+1)(1-\eta)+1\right]\frac{\sigma-1}{\sigma-\eta}}, \end{aligned} \tag{35}$$

*where for all* $k$

$$W_k(C_1, L_1, \Psi) = \left\{ \frac{\tilde{\beta}_k^{\frac{1}{1-\eta}} C_1^{(\sigma-1)\left(\frac{\epsilon_k}{1-\eta}+1\right)}}{(1-\phi_k)L_1} \left[1 - \frac{\Psi}{\phi_k^{-1} + N - 1}\right]\right\}^{\frac{1-\eta}{\sigma-\eta}}.$$

*Proof.* Eq. (33) has a unique solution $C_1$. Given this $C_1$ and $w_n \in (\underline{w_n}, \lambda_n]$, I let $L_n/L_1 = g_1^n(w_n)$ be the solution to eq. (34) and $L_n/L_1 = g_2^n(w_n)$ be that to eq. (35). I show that there exists unique $w_n \in (\underline{w_n}, \lambda_n)$ that solves $g_1^n(w_n) = g_2^n(w_n)$ by the intermediate value theorem. Given Assumption 4, $\lim_{w_n\to\underline{w_n}} g_1^n(w_n) > 0 = \lim_{w_n\to\underline{w_n}} g_2^n(w_n)$. At $w_n = \lambda_n$, eqs. (33) and (34) imply

$$\min_k \left\{ (g_1^n(\lambda_n))^{\gamma(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)-1} \lambda_n^{-\sigma}\right\} \le 1 \implies g_1^n(\lambda_n) < 1. \tag{36}$$

As for $g_2^n(\lambda_n)$, eq. (35) implies

$$1 \le \lambda_n^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta} \max_k \left(g_2^n(\lambda_n)\right)^{\left[\gamma(\frac{\epsilon(k)}{1-\eta}+1)(1-\eta)+1\right]\frac{\sigma-1}{\sigma-\eta}} \implies g_2^n(\lambda_n) > 1.$$

Therefore, $g_1^n(\lambda_n) < g_2^n(\lambda_n)$, and there exists an intersection at $w_n \in (\underline{w_n}, \lambda_n)$ by the intermediate value theorem because $g_1^n$ and $g_2^n$ are continuous on $(\underline{w_n}, \lambda_n]$. Further, the intersection is unique because $g_1^n(w_n)$ and $g_2^n(w_n)$ decreases and increases, respectively, in $w_n \in (\underline{w_n}, \lambda_n]$. It also follows that $0 < g_1^n(w_n) = g_2^n(w_n) < g_2^n(\lambda_n)$. Eq. (35) implies

$$1 \geq \lambda_n^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta} \min_k \left(g_2^n(\lambda_n)\right)^{\left[\gamma(\frac{\epsilon(k)}{1-\eta}+1)(1-\eta)+1\right]\frac{\sigma-1}{\sigma-\eta}}.$$

It follows that

$$\frac{L_n}{L_1} < g_2^n(\lambda_n) \leq \lambda_n^{\frac{1-\frac{1}{1-\eta}}{\gamma(\frac{\min_k \epsilon(k)}{1-\eta}+1)+\frac{1}{1-\eta}}} = \bar{\ell}_n.$$

This completes the proof of Lemma 9. □

Given Lemma 9, I let $g_3$ map $(L_1, \Psi) \in [L/\sum_{n\in\mathcal{N}} \bar{\ell}_n, L] \times [1 + \max_k \phi_k(N-1), N]$ to $(L_1', \Psi')$ such that

$$L_1' = \frac{L}{\sum_{n\in\mathcal{N}}(L_n/L_1)}, \quad \Psi' = \sum_{n\in\mathcal{N}} \left(\frac{w_n}{\lambda_n}\right)^\sigma,$$

where $L_n/L_1$ and $w_n$ form the unique solution in Lemma 9. Lemma 9 implies $\frac{1}{\sum_{n\in\mathcal{N}}\bar{\ell}_n} < L_1' < L$ and $1 + \max_k \phi_k(N-1) < \Psi' < N$. Thus, $g_3$ is a map from $[L/\sum_{n\in\mathcal{N}} \bar{\ell}_n, L] \times [1 + \max_k \phi_k(N-1), N]$ to itself, and it has a fixed point by Brouwer's fixed point theorem. Given the fixed point $(L_1, \sum_n (w_n/\lambda_n)^\sigma)$, the solution to the system of equations in Lemma 9 provide $C_1$, $\{L_n/L_1,\ w_n\}_{n=2\ldots N}$ satisfying eq. (23) for $n = 1, \ldots, N$ and (32) for $n = 2, \ldots, N$, and $\sum_{n\in\mathcal{N}} L_n = L$. □

*Proof of Lemma 4.* Let city $n$ be the more productive city ($\lambda_n > \lambda_i$). Equilibrium conditions (23) and (32) can be rewritten as

$$1 = \sum_{k\in\mathcal{K}} W_k(C_n, L_n, \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma), \tag{37}$$

$$1 = \sum_{k\in\mathcal{K}} W_k(C_n, L_n, \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma)$$
$$\cdot \left\{ \left(\frac{L_i}{L_n}\right)^{\gamma(\sigma-1)\left(\frac{\epsilon(k)}{1-\eta}+1\right)-1} \left(\frac{\lambda_i}{\lambda_n}\right)^{-\sigma} \left[ \frac{\phi_k^{-1} + N - 1 - (w_i/\lambda_i)^{-\sigma} \sum_{j\in\mathcal{N}} \left(\frac{w_j}{\lambda_j}\right)^\sigma}{\phi_k^{-1} + N - 1 - (w_n/\lambda_n)^{-\sigma} \sum_{j\in\mathcal{N}} \left(\frac{w_j}{\lambda_j}\right)^\sigma} \right] \right\}^{\frac{1-\eta}{\sigma-\eta}}, \tag{38}$$

$$1 = \left(\frac{w_i}{\lambda_i} / \frac{w_n}{\lambda_n}\right)^{1-2\sigma} \left(\frac{\lambda_i}{\lambda_n}\right)^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta} \sum_{k\in\mathcal{K}} X_k(C_n, L_n, \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma)$$
$$\cdot \left( \frac{\phi_k^{-1} + N - 1 - (w_n/\lambda_n)^{-\sigma} \sum_{j\in\mathcal{N}} \left(\frac{w_j}{\lambda_j}\right)^\sigma}{\phi_k^{-1} + N - 1 - (w_i/\lambda_i)^{-\sigma} \sum_{j\in\mathcal{N}} \left(\frac{w_j}{\lambda_j}\right)^\sigma} \right)^{\frac{\sigma-1}{\sigma-\eta}} \left(\frac{L_i}{L_n}\right)^{\left[\gamma(\frac{\epsilon(k)}{1-\eta}+1)(1-\eta)+1\right]\frac{\sigma-1}{\sigma-\eta}}, \tag{39}$$

where functions $W_k$ and $X_k$ are defined in the proof of Lemma 3. Applying the same steps as the proof of Lemma (9) lead to $[((\max \phi_k)^{-1} + N - 1)^{-1} \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma]^{1/\sigma} < w_i/\lambda_i < w_n/\lambda_n$. □

*Proof of Lemma 5.* I prove $L_1 > L_i$ for all $i \geq 2$ by contradiction. Suppose $L_1 \leq L_i$ in an equilibrium. Then, eqs. (37) and (38) where $n = 1$ imply

$$\min_k \lambda_i^{-\sigma} \frac{\phi_k^{-1} + N - 1 - (w_i/\lambda_i)^{-\sigma} \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma}{\phi_k^{-1} + N - 1 - \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma} \leq 1$$
$$\iff \left(\frac{w_i}{\lambda_i}\right)^{-\sigma} \geq (1 - \lambda_i^\sigma) \frac{\left[(\max \phi_k)^{-1} + N - 1\right]}{\sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma} + \lambda_i^\sigma, \tag{40}$$

where the second inequality used $w_i/\lambda_i < 1$. Further, eq. (39) and $\sum_{k\in\mathcal{K}} X_k(C_1, L_1, \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma) = 1$ imply

$$1 \geq \left(\frac{w_i}{\lambda_i}\right)^{1-2\sigma} \lambda_i^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta} \min_k \left( \frac{\phi_k^{-1} + N - 1 - \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma}{\phi_k^{-1} + N - 1 - (w_i/\lambda_i)^{-\sigma} \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma} \right)^{\frac{\sigma-1}{\sigma-\eta}}$$
$$> \lambda_i^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta} \left( \frac{(\min \phi_k)^{-1} + N - 1 - \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma}{(\min \phi_k)^{-1} + N - 1 - (w_i/\lambda_i)^{-\sigma} \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma} \right)^{\frac{\sigma-1}{\sigma-\eta}} \quad (\because w_i/\lambda_i < 1.)$$

Substituting eq. (40) for $(w_i/\lambda_i)^{-\sigma}$ yields

$$
\begin{aligned}
1 &> \lambda_i^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta}\left(\frac{(\min\phi_k)^{-1}+N-1-\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^{\sigma}}{(\min\phi_k)^{-1}-(\max\phi_k)^{-1}+\lambda_i^{\sigma}\left[(\max\phi_k)^{-1}+N-1-\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^{\sigma}\right]}\right)^{\frac{\sigma-1}{\sigma-\eta}} \\
&> \lambda_i^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta}\left(\frac{(\min\phi_k)^{-1}-1}{(\min\phi_k)^{-1}-(\max\phi_k)^{-1}+\lambda_i^{\sigma}\left[(\max\phi_k)^{-1}-1\right]}\right)^{\frac{\sigma-1}{\sigma-\eta}} \qquad (\because \sum_{j\in\mathcal{N}}(w_j/\lambda_j)^{\sigma}<N) \\
&= \lambda_i^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta}\left(\frac{1+\frac{(\max\phi_k)^{-1}-1}{(\min\phi_k)^{-1}-(\max\phi_k)^{-1}}}{1+\lambda_i^{\sigma}\left[\frac{(\max\phi_k)^{-1}-1}{(\min\phi_k)^{-1}-(\max\phi_k)^{-1}}\right]}\right) > \lambda_i^{\left(\frac{\sigma-1}{\sigma-\eta}\right)\eta}\left(\frac{1+\frac{1-\lambda_i^{\eta}}{\lambda_i^{\eta}-\lambda_i^{\sigma}}}{1+\lambda_i^{\sigma}\frac{1-\lambda_i^{\eta}}{\lambda_i^{\eta}-\lambda_i^{\sigma}}}\right)^{\frac{\sigma-1}{\sigma-\eta}} = 1.
\end{aligned}
$$

This is a contradiction. Thus, $L_1 > L_i$ for all $i \geq 2$. □

*Proof of Proposition 5.* The sectoral expenditure share ratio follows from eq. (12) as

$$
\begin{aligned}
\frac{s_{nk}}{s_{ik}}/\frac{s_{n\ell}}{s_{i\ell}} = \Bigg\{ &\left[1+\frac{(w_i/\lambda_i)^{-\sigma}-(w_n/\lambda_n)^{-\sigma}}{(1/\phi_k+N-1)/\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^{\sigma}-(w_i/\lambda_i)^{-\sigma}}\right] \\
&\cdot\left[1+\frac{(w_i/\lambda_i)^{-\sigma}-(w_n/\lambda_n)^{-\sigma}}{(1/\phi_\ell+N-1)/\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^{\sigma}-(w_i/\lambda_i)^{-\sigma}}\right]^{-1}\Bigg\}^{\frac{1-\eta}{\sigma-\eta}}.
\end{aligned}
$$

Given the same amenity level, $w_n > w_i$ implies $w_n/\lambda_n > w_i/\lambda_i$ following Proposition (4). Subsequently, the product inside the curly brackets is greater than one. □

*Proof of Lemmas 2 and 6.* Multiplying eq. (10) by $\mu_{nk}/(L_n w_n \sum_{\ell\in\mathcal{K}}\mu_{n\ell}s_{n\ell})$ yields

$$
\begin{aligned}
\frac{\mu_{nk}x_{nk}}{\sum_{\ell\in\mathcal{K}}\mu_{n\ell}s_{n\ell}} &= \frac{(1+\mu_{nk})\mu_{nk}s_{nk}}{\sum_{\ell\in\mathcal{K}}\mu_{n\ell}s_{n\ell}} - \frac{\mu_{nk}}{\sum_{\ell\in\mathcal{K}}\mu_{n\ell}s_{n\ell}}\frac{(w_n/\lambda_n)^{\sigma}}{(\phi_k^{-1}+N-1)L_n w_n}\sum_{i\in\mathcal{N}}\frac{L_i w_i}{(w_i/\lambda_i)^{\sigma}}(1+\mu_{ik})s_{ik} \\
&= \frac{(1+\mu_{nk})\mu_{nk}s_{nk}}{\sum_{\ell\in\mathcal{K}}\mu_{n\ell}s_{n\ell}} - \frac{\mu_{nk}}{\sum_{\ell\in\mathcal{K}}\mu_{n\ell}s_{n\ell}}\frac{(w_n/\lambda_n)^{\sigma}}{L_n w_n}\sum_{i\in\mathcal{N}}\frac{L_i w_i \mu_{ik}}{\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^{\sigma}}s_{ik},
\end{aligned}
$$

where the second equality follows from the definition of $\mu_{nk}$ (9). Substituting eq. (11) for $\sum_{j\in\mathcal{N}}(w_j/\lambda_j)^{\sigma}$

yields

$$\frac{\mu_{nk}x_{nk}}{\sum_{\ell\in\mathcal{K}}\mu_{n\ell}s_{n\ell}} = \frac{(1+\mu_{nk})\mu_{nk}s_{nk}}{\sum_{\ell\in\mathcal{K}}\mu_{n\ell}s_{n\ell}} - \mu_{nk}\sum_{i\in\mathcal{N}}\frac{L_i w_i \mu_{ik}}{\sum_{j\in\mathcal{N}}\left(\sum_{\ell\in\mathcal{K}}\mu_{j\ell}s_{j\ell}\right)L_j w_j}s_{ik}$$
$$= \frac{(1+\mu_{nk})\mu_{nk}s_{nk}}{\sum_{\ell\in\mathcal{K}}\mu_{n\ell}s_{n\ell}} - \mu_{nk}\sum_{i\in\mathcal{N}}\frac{\left(\sum_{\ell\in\mathcal{K}}\mu_{j\ell}s_{j\ell}\right)L_i w_i}{\sum_{j\in\mathcal{N}}\left(\sum_{\ell\in\mathcal{K}}\mu_{j\ell}s_{j\ell}\right)L_j w_j}\frac{\mu_{ik}s_{ik}}{\left(\sum_{\ell\in\mathcal{K}}\mu_{j\ell}s_{j\ell}\right)}.$$

This completes the proof of Lemma 6. Lemma 2 is a special case of $\mu_{nk} = \mu_n$, and immediately follows from this. □

*Proof of Lemma 7.* I prove that the higher-income city is more productive than the lower-wage city by contradiction. Suppose the higher-income city has a lower factor price; that is, $w_n > w_i$, and $w_n/\lambda_n < w_i/\lambda_i$. Then, it implies $\lambda_n > \lambda_i$, which in turn implies $w_n/\lambda_n > w_i/\lambda_i$, according to the city-level labor demand eq. (24). This is a contradiction; therefore, $w_n/\lambda_n > w_i/\lambda_i$. Subsequently, eq. (24) implies $\lambda_n > \lambda_i$.

Dividing the expenditure share ratio for sector $k$ by that of sector $\ell$ yields

$$\frac{s_{nk}}{s_{ik}}\Big/\frac{s_{n\ell}}{s_{i\ell}} = \left(\frac{C_n}{C_i}\right)^{\frac{\epsilon(k)-\epsilon(\ell)}{1-\eta}(\sigma-1)\frac{1-\eta}{\sigma-\eta}} = \left(\frac{L_n^\gamma/a_n}{L_i^\gamma/a_i}\right)^{\frac{\epsilon(k)-\epsilon(\ell)}{1-\eta}(\sigma-1)\frac{1-\eta}{\sigma-\eta}} = \left(\frac{a_i}{a_n}\right)^{\frac{\epsilon(k)-\epsilon(\ell)}{1-\eta}(\sigma-1)\frac{1-\eta}{\sigma-\eta}},$$

where the second equality follows from (4). Proposition (2) implies that when two cities have the same population and one has a higher productivity, the other must have a higher level of amenities. Thus, $a_i/a_n > 1$. □

# C Equilibrium Stability and Uniqueness

This appendix shows the stability and uniqueness of an equilibrium for the case of two cities and common tradabilities. I start by defining a stable equilibrium in this model of two cities.

**Definition 2** (Stable Equilibrium)**.** A competitive equilibrium in the model of two cities is a stable equilibrium if and only if

$$\frac{d\left(C_1 a_1/C_2 a_2\right)^{1/\gamma}}{d\left(L_1/L_2\right)} < 1.$$

This inequality guarantees that, with the migration of infra-marginal agents who are indifferent between two cities ($C_1 a_1 \delta(\zeta, 2) = C_2 a_2 \delta(\zeta, 2)$), the expanding city would not experience a sufficient relative gain in real income to support the post-migration population allocation. Given this definition, I obtain Proposition 6.

**Proposition 6** (Uniqueness and Stability)**.** *Given Assumptions 1 and 2, common tradability, and* $\mathcal{N} = \{1, 2\}$*, the equilibrium in which all sectors have non-zero production in all cities is unique and stable.*

*Proof.* I introduce new variables, $V_1$ and $V_2$, defined as

$$V_1 = a_1 C_1 L_1^{-\gamma} \text{ and } V_2 = a_2 C_2 L_2^{-\gamma}.$$

Subsequently, the condition for a stable equilibrium becomes

$$\begin{aligned} &\frac{d\,(V_1/V_2)^{1/\gamma}\,(L_1/L_2)}{d\,(L_1/L_2)} < 1 \\ \Longrightarrow\ &\frac{d\,(V_1/V_2)}{d\,(L_1/L_2)} < 0, \end{aligned}$$

where I used $V_1 = V_2$ at an equilibrium. Given $V_1$, $V_2$, and normalization of $\lambda_1 = a_1 = w_1 = 1$, the system of equations that determines real income in the two cities can be rewritten as follows:

$$1 = \sum_{k \in \mathcal{K}} \left\{ \frac{\tilde{\beta}_k^{\frac{1}{1-\eta}} (V_1)^{(\sigma-1)\left(\frac{\epsilon_k}{1-\eta}+1\right)}}{1-\phi} L_1^{\gamma(\sigma-1)\left(\frac{\epsilon_k}{1-\eta}+1\right)-1} \left[1 - \frac{\sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma}{\phi^{-1} + N - 1}\right] \right\}^{\frac{1-\eta}{\sigma-\eta}}. \tag{41}$$

$$1 = \sum_{k \in \mathcal{K}} \left\{ \frac{\tilde{\beta}_k^{\frac{1}{1-\eta}} \left(\frac{V_2}{a_2}\right)^{(\sigma-1)\left(\frac{\epsilon_k}{1-\eta}+1\right)}}{1-\phi} L_2^{\gamma(\sigma-1)\left(\frac{\epsilon_k}{1-\eta}+1\right)-1} \left[1 - \frac{\sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma}{(\phi^{-1} + N - 1)(w_2/\lambda_2)^\sigma}\right] \right\}^{\frac{1-\eta}{\sigma-\eta}}, \tag{42}$$

$$1 = \left[\frac{\phi^{-1} + N - 1 - \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma}{\phi^{-1} + N - 1 - (w_2/\lambda_2)^{-\sigma} \sum_{j\in\mathcal{N}} (w_j/\lambda_j)^\sigma}\right] \lambda_2 \left(\frac{w_2}{\lambda_2}\right)^{1-2\sigma} \frac{L_2}{L_1}. \tag{43}$$

This system maps $(L_1, L_2)$ to $(V_1, V_2)$, and $(L_1, L - L_1)$ that makes $V_1 = V_2$ is an equilibrium. We know from Proposition 1 that there exits such $(L_1, L - L_1)$. Eq. 43 implies $\partial w_2 / \partial \frac{L_1}{L_2} < 0$. Subsequently, eqs. 41 and 42 with $L_1 + L_2 = L$ imply

$$\frac{dV_1}{d\,(L_1/L_2)} < 0, \quad \frac{dV_2}{d\,(L_1/L_2)} > 0.$$

Thus, $V_1$ decreases and $V_2$ increases with $L_1/L_2$, implying the intersection ($V_1 = V_2$) is unique. Further, these signs imply

$$\frac{d\left(V_1/V_2\right)}{d\left(L_1/L_2\right)} < 0.$$

Therefore, it is a stable equilibrium. $\square$